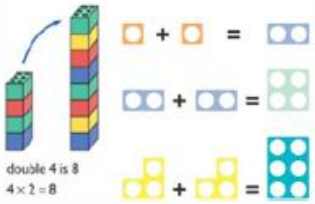
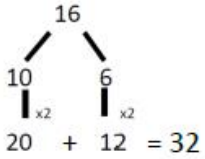
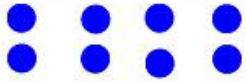
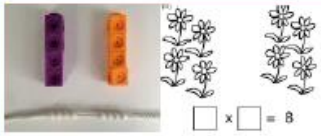
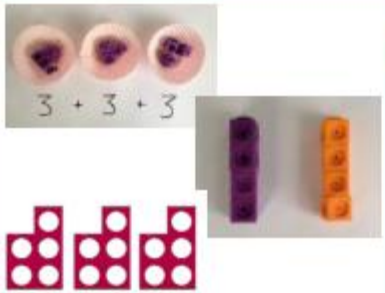
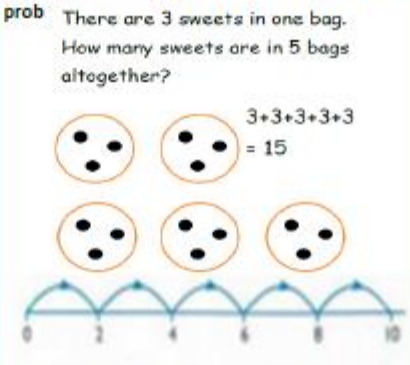
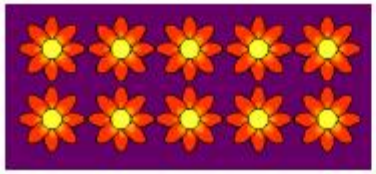
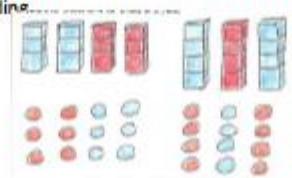
Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count

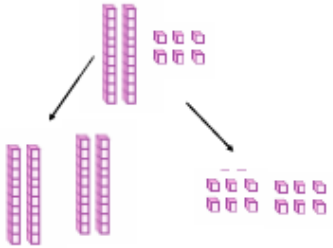
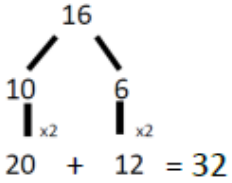
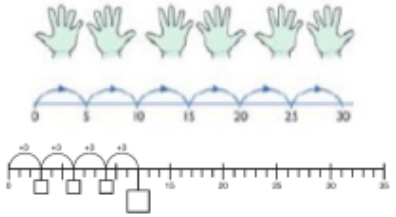
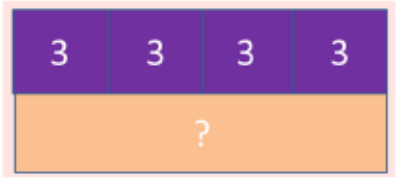
Y1 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling 	Draw pictures to show how to double numbers	Partition a number and then double each part before recombining it back together. 
Counting in multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting.	 Children make representations to show counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Making equal groups and counting the total	 Use manipulatives to create equal groups.	Draw to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8

Y1 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	 Use different objects to add equal groups	Use pictorial including number lines to solve prob 	Write addition sentences to describe objects and pictures.
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding 	3 x 2 = 6 2 x 5 = 10

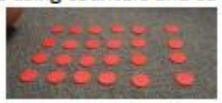
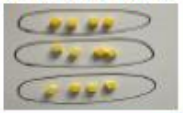
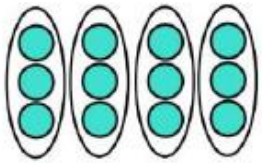
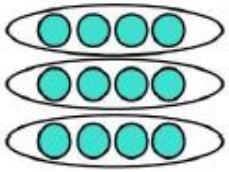
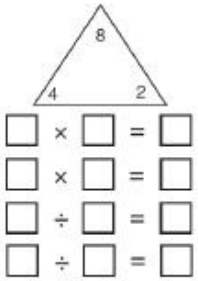
Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times. number line

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters.  40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together.  16 10 (x2) 6 (x2) 20 + 12 = 32
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models. 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40 ?	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.   3 3 3 3 ?	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$

Y2 MULTIPLICATION X

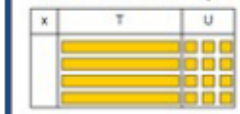
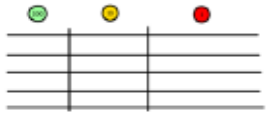
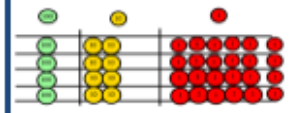
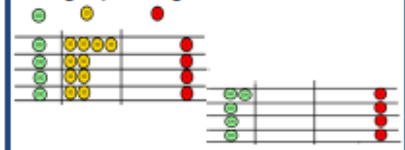
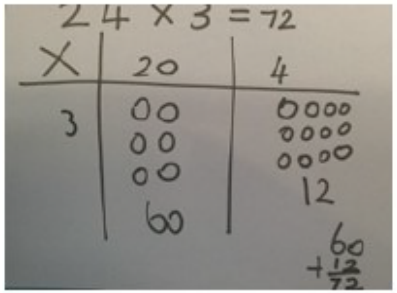
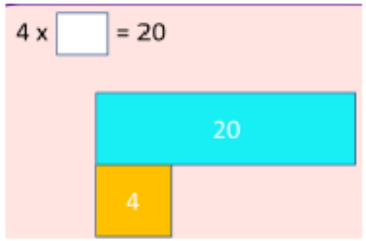
Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times. number line

Y2 MULTIPLICATION

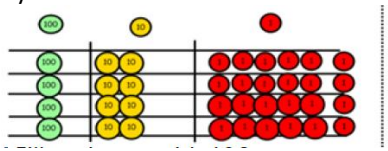
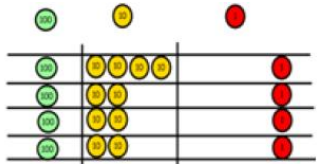
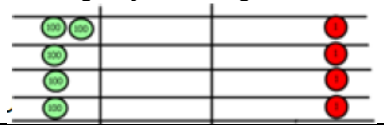
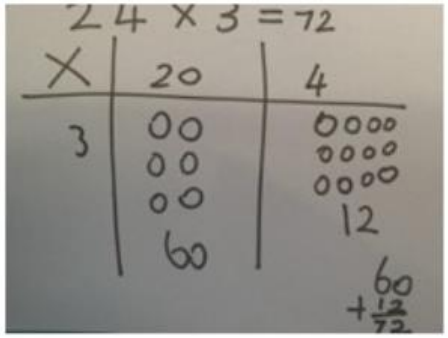
Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon.  Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer. 	Use representations of arrays to show different calculations and explore commutativity.  	12 = 3 × 4 12 = 4 × 3 Use an array to write multiplication sentences and reinforce repeated addition. 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*		 	2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times. number line partition, grid method, multiple, product, tens, units, value, inverse, multiple

Y3 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method	Show the links with arrays to first introduce the grid method 4 rows of 10 4 rows of 3 Move onto base ten to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Calculations 4 x 126 Fill each row with 126  Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed  Then you have your answer.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  Bar model are used to explore missing numbers  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. × | 30 | 5 7 | 210 | 35 210 + 35 = 245 Moving forward, multiply by a 2 digit number showing the different rows within the grid method. 10 | 8 10 | 100 | 80 3 | 30 | 24

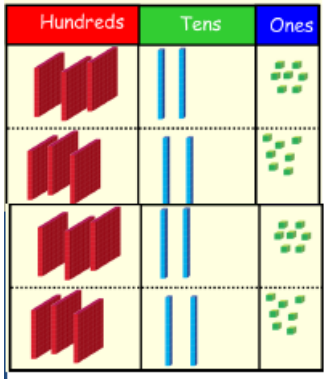
Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times. number line partition, grid method, multiple, product, tens, units, value, inverse, multiple **inverse**

Objective & strategy	Concrete	Pictorial	Abstract
Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Calculations 4 x 126 Fill each row with 126  Add up each column, starting with the ones making any exchanges needed 	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. **210 + 35 = 245**

×	30	5
7	210	35

Column Multiplication

Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642

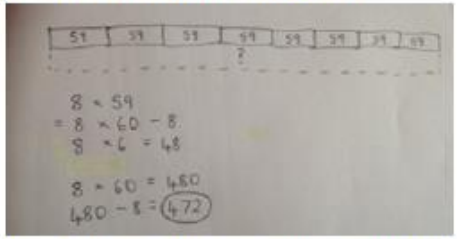


It is important at this stage that they always multiply the ones first.

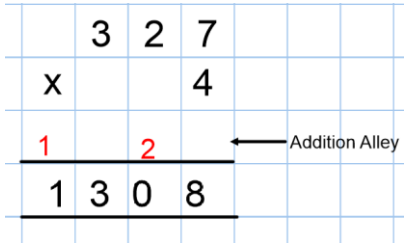
The corresponding long multiplication is modelled alongside

x	300	20	7
4	1200	80	28

The grid method my be used to show how this relates to a formal written method.

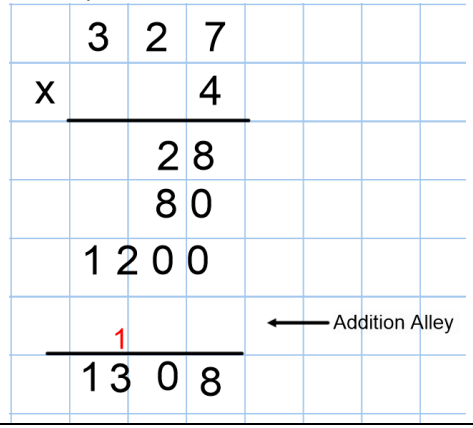


Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



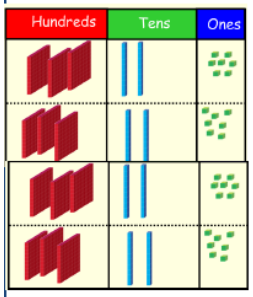
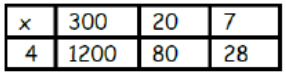
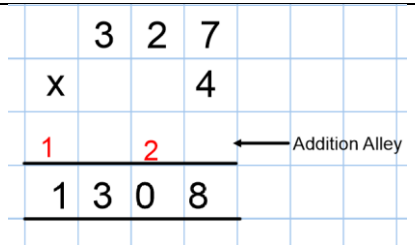
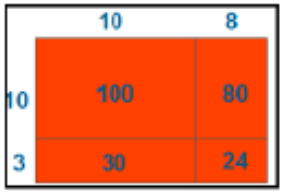
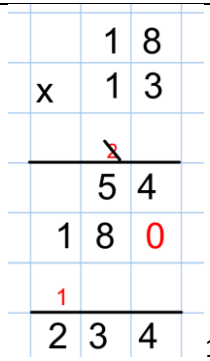
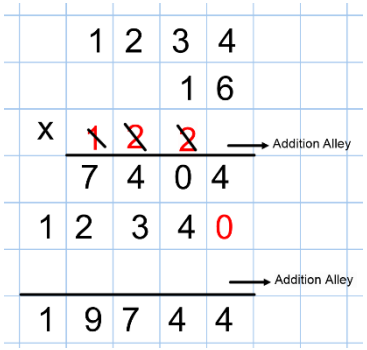
Compact method

If children can not at first move to the compact method, they may need to use expanded first. See below.

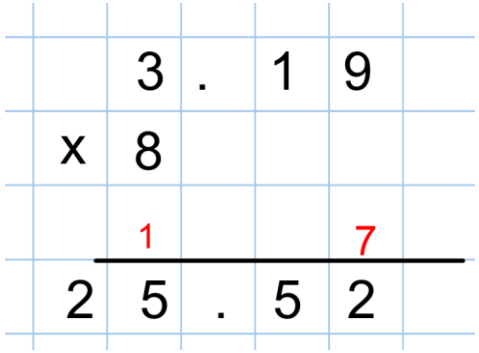


Y4 MULTIPLICATION

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times. number line partition, grid method, multiple, product, tens, units, value, inverse, multiple square, factor, integer, decimal, short/long multiplication, 'carry', quotient, prime number, prime factors, composite number (non-prime)

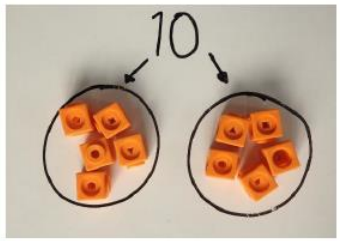
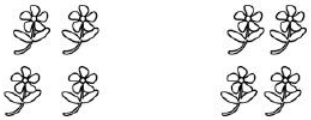
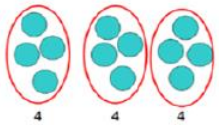
Objective & strategy	Concrete	Pictorial	Abstract
Column Multiplication for 3 and 4 digits x 1 digit.	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside	 	
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	 Continue to use bar modelling to support problem solving	 Needing two addition alleys. The first one, the numbers are crossed off 

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times. number line partition, grid method, multiple, product, tens, units, value, inverse, multiple tenths, hundredths, decimal, common factor, common multiple

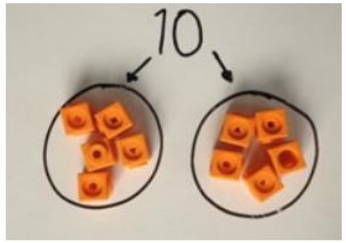
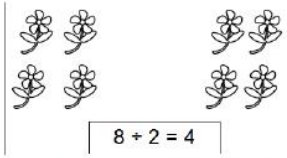
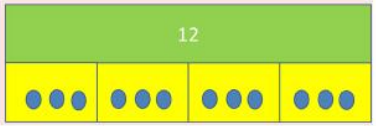
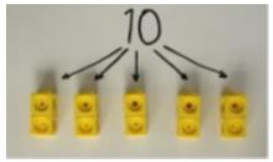
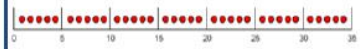
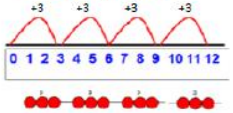
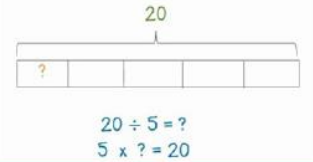
Objective & strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 

Y6 MULTIPLICATION X

Key Vocabulary: groups of, lots of, array, altogether, multiply, count, share, share equally, one each, two each…, group, groups of, lots of

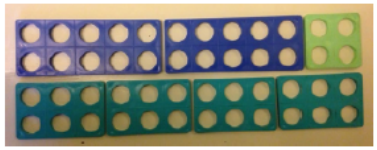
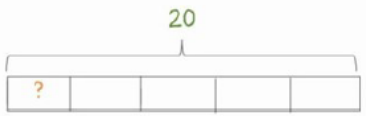
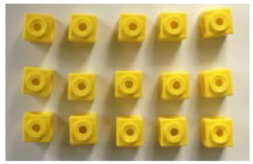
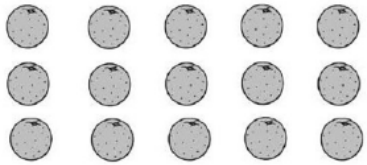
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing *Use Gordon ITPs for modelling*	  I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 shared between 2 is 4 Sharing:  12 shared between 3 is 4	12 shared between 3 is 4

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, **share, share equally, one each, two each…, group, equal groups of,** multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times… **divide, divided by, divided into, division, grouping, number line, left, left over**

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	12 ÷ 3 = 4
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  **12 ÷ 3 = 4** Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

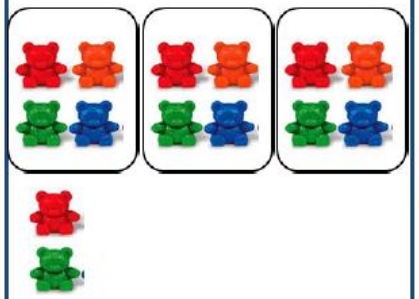
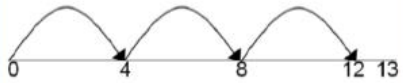
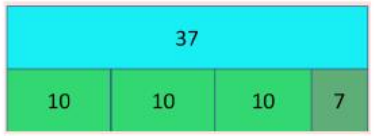
Y2 DIVISION ÷

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times..., share, share equally, one each, two each..., group, equal groups of, divide, divided by, divided into, division, grouping, number line, left, left over, partition, grid method, multiple, product, tens, units, value, inverse, short division, 'carry', remainder, multiple

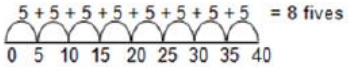
Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 96 ÷ 3 = 32 	Continue to use bar modelling to aid solving division problems.  20 ÷ 5 = ? 5 x ? = 20	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

Y3 DIVISION ÷

Y3 DIVISION ÷

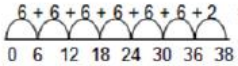
Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. Use bar models to show division with remainders.  	Complete written divisions and show the remainder using r.

Example without remainder:
40 ÷ 5
Ask "How many 5s in 40?"



= 8 fives

Example with remainder:
38 ÷ 6



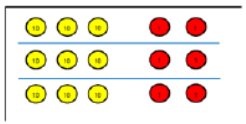
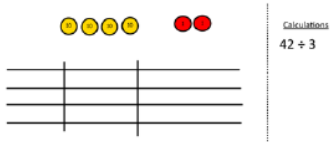
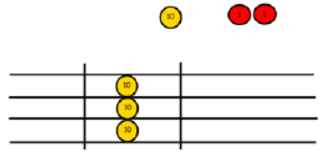
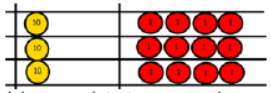
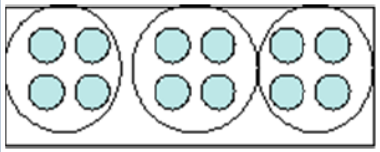
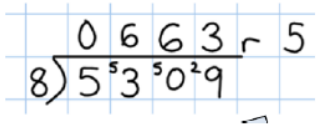
= 6 sixes with a remainder of 2

For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times as big as, once, twice, three times… partition, grid method, total, multiple, product, sets of, share, share equally, one each, two each…, group, equal groups of, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, "carry", remainder, multiple, inverse, divisible by, factor

Y4-6

DIVISION ÷

Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	96 ÷ 3 Tens Units 3 2 3  Use place value counters to divide using the bus stop method alongside  Calculations 42 ÷ 3 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. 2 1 8 3 4 8 7 2 Move onto divisions with a remainder. 8 6 r 2 3 5 4 3 2 Finally move into decimal places to divide the total accurately. 1 4 . 6 16 21 3 5 5 1 1 . 0  0 6 6 3 r 5 8 5 3 5 0 2 9

Long Division

Step 1—a remainder in the ones

```
   h t o
   0 4 1 R1
4 ) 1 6 5
```

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

```
    th h t o
    0 4 0 0 R7
8 ) 3 2 0 7
```

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

```
   h t o
   0 6 1
4 ) 2 4 7
   - 4
     3
```

When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3.

Check: 4 × 61 + 3 = 247

```
    th h t o
    0 4 0 2
4 ) 1 6 0 9
   - 8
     1
```

When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

Y6 DIVISION ÷

Long Division

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2)5 8 Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	t o 2 2)5 8 - 4 1 To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	t o 2 9 2)5 8 - 4 ↓ 1 8 Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2)5 8 - 4 1 8 Divide 2 into 18. Place 9 into the quotient.	t o 2 9 2)5 8 - 4 1 8 - 1 8 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	t o 2 9 2)5 8 - 4 1 8 - 1 8 0 The division is over since there are no more digits in the dividend. The quotient is 29.

Y6

DIVISION ÷

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)2 7 8 Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	h t o 1 2)2 7 8 - 2 0 Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	h t o 1 8 2)2 7 8 - 2 ↓ 0 7 Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 2)2 7 8 - 2 0 7 Divide 2 into 7. Place 3 into the quotient.	h t o 1 3 2)2 7 8 - 2 0 7 - 6 1 Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	h t o 1 3 2)2 7 8 - 2 0 7 - 6 1 8 Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8 Divide 2 into 18. Place 9 into the quotient.	h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8 - 1 8 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8 - 1 8 0 There are no more digits to drop down. The quotient is 139.