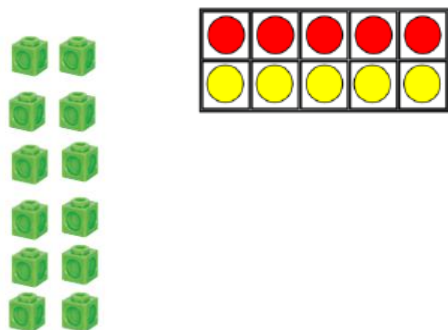


Key Vocabulary: whole, parts of a whole fraction, one whole, half, quarter, three-quarters

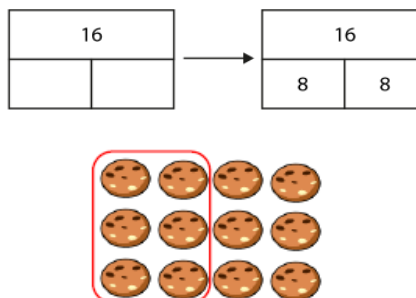
Concrete

Find $\frac{1}{2}$ of ___
Find half of ___



Pictorial

Find $\frac{1}{2}$ of 16
Find half of 16



Abstract

Find $\frac{1}{2}$ of 16
Find half of 16

$$\frac{1}{2} \text{ of } 4 = \square$$

$$\frac{1}{2} \text{ of } 6 = \square$$

$$\frac{1}{2} \text{ of } 8 = \square$$

Model	Say	Write	Notation
 one-half	<i>'The apple has been divided...'</i>	Write the division bar.	$\frac{1}{2}$
	<i>'...into 2 equal parts...'</i>	Write '2' as the denominator.	
	<i>'...and we have 1 of the parts.'</i>	Write '1' as the numerator.	



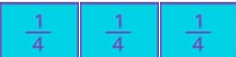


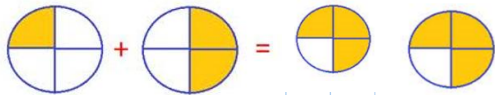
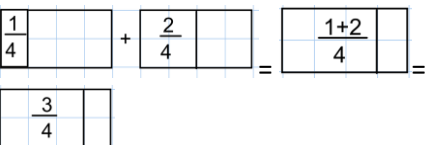
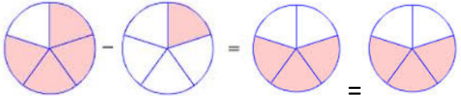
A key concept is for teachers to use the vocabulary in Key Stage 1 of 'part' and 'whole' and to always refer back to this vocabulary ready for Key Stage 2.

Model	Say	Write	Notation
 one-third	<i>'The rectangle has been divided...'</i>	Write the division bar.	$\frac{1}{3}$
	<i>'...into 3 equal parts...'</i>	Write '3' as the denominator.	
	<i>'...and 1 of the parts is shaded.'</i>	Write '1' as the numerator.	

Model	Say	Write	Notation
 one-quarter	<i>'The strawberries have been divided...'</i>	Write the division bar.	$\frac{1}{4}$
	<i>'...into 4 equal parts...'</i>	Write '4' as the denominator.	
	<i>'...and 1 of the parts is circled.'</i>	Write '1' as the numerator.	

Y3 FRACTIONS

Key Vocabulary: **fraction**, **one whole**, **half**, **quarter**, **three-quarters**, **add**, **subtract**, **equal parts of a whole**, **numerator**, **denominator**

Objective & strategy	Concrete	Pictorial	Abstract
<p>Add and subtract fractions with the same denominator within one whole (e.g. $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$)</p>	<p> +  =</p> <p></p> <p>Introduce stem sentence 'when the denominators are the same, normal rules of arithmetic apply.'</p> <p></p> <p>$\frac{5}{10} + \frac{3}{10} = \frac{5+3}{10} = \frac{8}{10}$</p> <p>Subtraction</p> <p></p> <p>Use a variety of resources such as number rods, paper strips, equivalence circles, cards to model what happens when you add and subtract fractions with the same denominator.</p>	<p></p> <p>$\frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} = \frac{3}{4}$</p> <p>Bar model</p> <p></p> <p>$\frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} = \frac{3}{4}$</p> <p>Subtraction</p> <p></p> <p>$\frac{4}{5} - \frac{1}{5} = \frac{4-1}{5} = \frac{3}{5}$</p>	<p>$\frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} = \frac{3}{4}$</p> <p>Subtraction</p> <p>$\frac{4}{5} - \frac{1}{5} = \frac{4-1}{5} = \frac{3}{5}$</p>

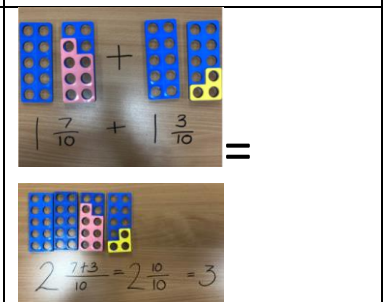
Key Vocabulary: **fraction, whole-one/s, half, quarter, three-quarters, numerator, denominator, add, subtract, equal parts of a whole, mixed number, equivalence, equivalent**

Objective & strategy

Add and subtract fractions with the same denominator

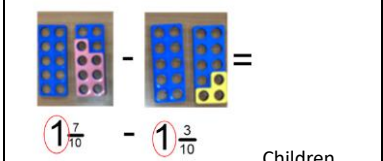
Continue to develop addition and subtraction of fractions as shown in Year 3 but beyond the whole one, using lots of practical resources such as number rods, equivalence circles, cards, etc to ensure conceptual understanding.

Concrete



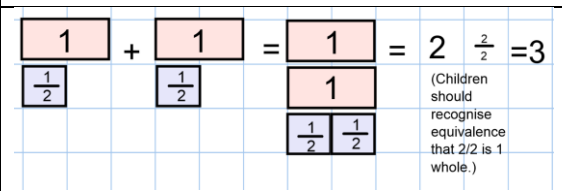
Children should understand the vocabulary parts and wholes. Children should deal with the wholes first then the parts.

Subtractions



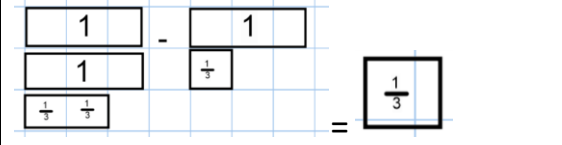
Children deal with the wholes first then the parts. Only exchanging if the fraction goes below a whole.

Pictorial



Bar model
Children will add the whole ones first. **1 whole + 1 whole = 2 wholes.**
Then, add the parts
 $\frac{1}{2} + \frac{1}{2} = 1$ whole.
Then use equivalence to recognise that $\frac{2}{2} = 1$ whole.
The same concept for subtraction calculations is to be followed.

Subtractions



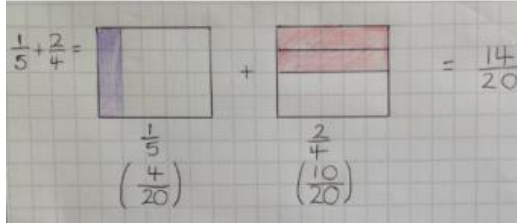
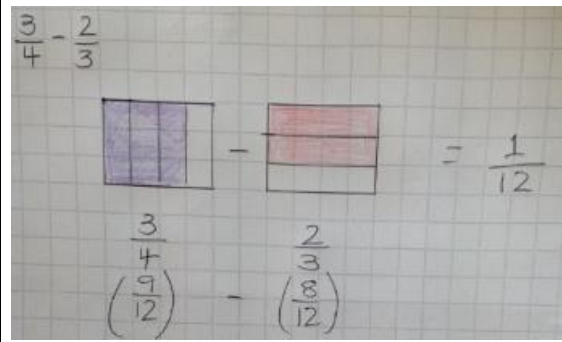
Abstract

$$1 \frac{1}{2} + 1 \frac{1}{2} = 2 \frac{2}{2} = 3$$

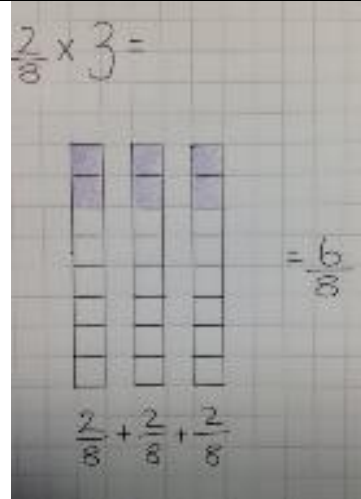
Subtractions

$$2 \frac{2}{3} - 1 \frac{1}{3} = 1$$

Key Vocabulary: fraction, whole-one/s, half, quarter, three-quarters, numerator, denominator, add, subtract, equal parts of a whole, mixed number, equivalence, equivalent, array, model, improper fraction, multiples, common denominator

Objective & strategy	Concrete	Pictorial	Abstract
<p>Add and subtract fractions with the same denominator and multiples of the same number</p>		<p>Addition</p>  <p>The grid is drawn 5x4 for both to show the common multiple.</p> <p>Subtraction</p> 	<p>Children use common multiples to find the common denominator.</p> $\frac{1^{x4}}{5^{x4}} + \frac{2^{x5}}{4^{x5}} = \frac{14}{20}$ $\frac{4}{20} + \frac{10}{20} = \frac{10+4}{20} = \frac{14}{20}$ <p>Children should try and simplify at the end of the calculation if possible.</p> $\frac{3^{x3}}{4^{x3}} + \frac{2^{x4}}{3^{x4}} = \frac{1}{12}$ $\frac{9}{12} + \frac{8}{12} = \frac{9-8}{12} = \frac{1}{12}$

Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams


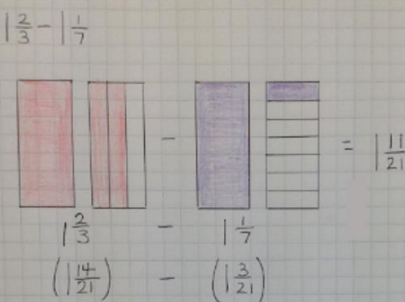
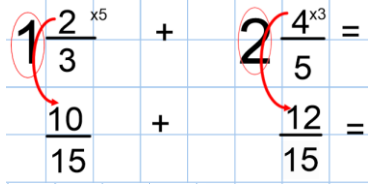


A similar model can be used for repeated addition for multiplication of fractions and mixed numbers by whole numbers

$\frac{3}{6}$	\times	7	$=$	$\frac{3 \times 7}{6}$	$=$	$\frac{21}{6}$

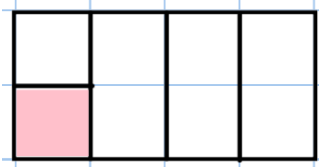
Children should then simplify into a mixed number.

Key Vocabulary: : fraction, whole-one/s, half, quarter, three-quarters, numerator, denominator, equal parts of a whole, mixed number, equivalence, equivalent, array, model, improper fraction, common denominator, multiples, multiply, repeated addition

Objective & strategy	Concrete	Pictorial	Abstract
<p>Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions</p> <p>Children should continually be taught to deal with the wholes first then the parts.</p>		<p>$1\frac{2}{3} + 2\frac{4}{5} = 3\frac{22}{15}$</p>  <p>$= 4\frac{7}{15}$</p> <p>Children should continually be taught to deal with the wholes first then the parts. Children simplify at the end .</p> <p>Subtraction</p>  <p>$\frac{2}{3} - \frac{1}{7} = \frac{11}{21}$</p> <p>When children are able to move on from the above model, they might encounter difficulties when using mixed numbers. For example, if it was $1\frac{1}{4} - \frac{1}{2}$ this would require the understanding that they would need to break the whole one down into quarters. To ensure that children do not become confused by this, they should be taught to convert the mixed numbers to improper fractions first and then converted back, once the calculation is done.</p>	<p>$1\frac{2}{3} + 2\frac{4}{5} =$</p>  <p>$1\frac{2}{3} = \frac{10}{15}$ $2\frac{4}{5} = \frac{12}{15}$</p> <p>$\frac{22}{15} = 1\frac{7}{15}$</p> <p>Children should continually be taught to deal with the wholes first then the parts.</p>

Multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$)

$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$



Step 1: draw a grid that shows both quarters and halves. At this point, only draw lines that show quarters of the grid because this is what is being multiplied. **Step 2:** Look at one of the quarters and shade half of it (because we are multiplying by a half). At this point children will need to know that when multiplying by a fraction, the number does not get bigger, it actually gets smaller. **Step 3:** Find the fraction that is shaded by drawing an imaginary line across the whole grid halving it also. How many equal parts are shaded? (Numerator) How many equal parts has the grid been divided into? (Denominator)

$$\frac{6}{10} \times \frac{3}{4} = \frac{6 \times 3}{10 \times 4} = \frac{18}{40}$$

Children should see from the pictorial explanation that the numerators are multiplied together, and the denominators are multiplied together.

Divide proper fractions by whole numbers (e.g. $\frac{1}{3} \div 2 = \frac{1}{6}$)

$$\frac{1}{3} \div 2 = \frac{1}{6}$$



Step 1: draw a grid that shows the dividend ($\frac{1}{3}$ in this case) and also the divisor (2 in this case).

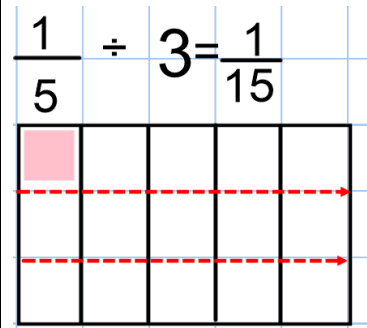
$$\frac{1}{3} \div 2 = \frac{1}{3 \times 2} = \frac{1}{6}$$

$$\frac{1}{5} \div 3 = \frac{1}{5 \times 3} = \frac{1}{15}$$

Draw lines down to show the dividend ($\frac{1}{3}$) but do not draw lines across to show the divisor at this point.

Step 2: look at one of the thirds and divide it by 2 (the divisor). Shade in one part of this.

Step 3: To find the answer, look at the fraction that is shaded by drawing an imaginary line across the whole grid, dividing it by 2. How many equal parts are shaded? (Numerator) How many equal parts has the grid been divided into? (Denominator). In this case the answer is $\frac{1}{6}$



Year 1

Addition and Subtraction

Video Clip: <https://www.youtube.com/watch?v=OkW1Y11tGxw&list=UUVb98bWNgEmk02R7enUrmFA>

Year 2

Addition and Subtraction

Video: <https://www.youtube.com/watch?v=mEHK mMapWGY>

Multiplication and Division

Multiple Representations of Multiplication

<https://www.youtube.com/watch?v=YPWmOVt8vgw&list=UUVb98bWNgEmk02R7enUrmFA>

The Commutative Law for Multiplication

<https://www.youtube.com/watch?v=VGkijVfnGYI&list=UUVb98bWNgEmk02R7enUrmFA>

Sharing and Grouping (whole class)

<http://vimeo.com/83485518>

Sharing and Grouping (pairs)

<http://vimeo.com/83485658>

Year 3

Addition and Subtraction

Videos:

Subtraction—teaching children to consider the most appropriate methods before calculating <http://www.youtube.com/watch?v=RCCLseBLBSo>

Introducing partitioned column subtraction method, from practical to written

<http://www.youtube.com/watch?v=dP8NIFLZzOg>

Multiplication and Division

Videos: (Y4 examples but can be adapted to suit Year 3)

Representing division with place value counters <http://vimeo.com/83485661>

Using place value counters and recording division <http://vimeo.com/83485662>

Year 4

Addition and Subtraction

Subtraction—teaching children to consider the most appropriate methods before calculating <http://www.youtube.com/watch?v=RCCLseBLBS0>

Introducing partitioned column subtraction method, from practical to written

<http://www.youtube.com/watch?v=dP8NIFLZzOg>

Moving to the compact column method of subtraction <http://www.youtube.com/watch?v=3ihxp2mqnhs>

Multiplication and Division

Multiplication – Lower Key Stage 2 (links to place value counters as shown in Y3) <http://vimeo.com/70319240>

Representing division with place value counters <http://vimeo.com/83485661>

Using place value counters and recording division <http://vimeo.com/83485662>

Year 5

Addition and Subtraction

Video:

Moving to the compact column method of subtraction <http://www.youtube.com/watch?v=3ihxp2mqnhs>

Multiplication and Division

Multiplication – Upper Key Stage 2 <http://vimeo.com/70318365>

Rapid Recall of Multiplication Facts https://www.youtube.com/watch?v=BcljRLZzMaw&list=PLQqF8sn28L9wjDm8uJEJcRCDDoY6raPE_&index=2

Year 6

Addition and Subtraction

See videos in Years 4 and 5 to show understanding of compact method

Multiplication and Division

Videos:

Rapid Recall of Multiplication Facts https://www.youtube.com/watch?v=BcljRLZzMaw&list=PLQqF8sn28L9wjDm8uJEJcRCDDoY6raPE_&index=2

Glossary of Terms

2-digit number – a number with 2 digits like 23, 45, 12 or 60

3-digit number – a number with 3 digits like 123, 542, 903 or 561

Addition facts – knowing that $1+1 = 2$ and $1+3 = 4$ and $2+5 = 7$. Normally we only talk about number facts with totals of 20 and under.

Array - An array is an arrangement of a set of numbers or objects in rows and columns –it is mostly used to show how you can group objects for repeated addition or subtraction.

Bead String/Bar – a string with (usually 100) beads on, grouped by colour in tens. The bead string is a good bridge between a number track and a number line as it maintains the cardinality of the numbers whilst beginning to develop the concepts of counting ‘spaces’ rather than objects.

Bridging – when a calculation causes you to cross a ‘ten boundary’ or a ‘hundred boundary’ e.g. $85 + 18$ will bridge 100.

Compact vertical – the name of the recommended written method for addition whereby the numbers are added in columns, 1s first then 10s and so on. Where the total exceeds 10, the ten 1s are exchanged for a 10 and written below the answer line. Sometimes referred to as ‘carrying’.

Concrete apparatus – objects to help children count and calculate– these are most often cubes (multilink) but can be anything they can hold and move including Cuisenaire rods, Dienes rods (hundreds, tens and units blocks), straws, Numicon, Place Value counters and much more.

Count all – when you add by counting all the items/objects e.g. to add 11 and 5 you would count out 11, then count out 5, then put them together and count them all to get 16.

Count on – when you add (or sometimes subtract) by counting onwards from a given number. E.g. to add 11 and 5 you would count on 5 from 11 i.e. 12, 13, 14, 15, 16

Decimal number – a number with a decimal point e.g. 2.34 (said as two point three four)

Decomposition – the name of the recommended written method for subtraction whereby the smaller number is subtracted from the larger, 1s first then 10s and so on. Where the subtraction cannot be completed as the second number is larger than the first, a 10 is exchanged for ten 1s to facilitate this. This is the traditional ‘borrowing’ form of column method, which is different to the ‘payback’ method.

Dienes Rods (or Base 10) – this is a set of practical equipment that represents the numbers to help children with place value and calculation. The Dienes rods show 1s, 10s, 100s and 1000s as blocks of cubes that children can then combine. Dienes rods do not break up so the child has to ‘exchange’ them for smaller or larger blocks where necessary.

Difference – the gap between numbers that is found by subtraction e.g. 7-5 can be read as ‘7 take away 5’ or as the ‘difference between 7 and 5’

Dividend – the number being divided in a calculation

Divisor – the smaller number in a division calculation.

Double – multiply a number by 2

Efficient Methods – the method(s) that will solve the calculation most rapidly and easily

Equals - is worth the same as (be careful not to emphasise the use of = to show the answer)

Exchanging – Swapping a ‘10’ for ten ‘1s’ or a ‘100’ for ten ‘10s’ or vice versa (used in addition and subtraction when ‘moving’ ‘ten’ or a ‘hundred’ from its column into the next column and splitting it up). Heavily relied upon for addition and subtraction of larger numbers. Skills in this can be built up practically with objects, then Dienes rods/base 10, then place value counters before relying on a solely written method.

Expanded Multiplication – a method for multiplication where each stage is written down and then added up at the end in a column

Factor – a number that divides exactly into another number, without remainder

Grid method – a method for multiplying two numbers together involving partitioning and multiplying each piece separately.

Grouping – an approach to division where the dividend is split into groups of the size of the divisor and the number of groups created are then counted.

Half - a number, shape or quantity divided into 2 equal parts

Halve – divide a number by 2

Integer - a whole number (i.e. one with no decimal point)

Inverse – the opposite operation. For example, addition is the inverse of subtraction and multiplication is the inverse of division.

Known Multiplication Facts – times tables and other number facts that can be recalled quickly to support with larger or related calculations e.g. if you know 4×7 then you also know 40×7 , 4×0.7 etc.

Long Division – formal written of division where the remainders are calculated in writing each time (extended version of short division)

Long Multiplication – formal written method of column multiplication

Multiple - a number which is an exact product of another number i.e. a number which is in the times table of another number

Number bonds – 2 numbers that add together to make a given total, e.g. 8 and 2 bond to 10 or 73 and 27 bond to 100

Number line – a line either with numbers or without (a blank numberline).

The number line emphasises the continuous nature of numbers and the existence of ‘in-between’ numbers that are not whole. It is based around the gaps between numbers.

Children use this tool to help them count on or count back for addition or subtraction. As they get older, children will count in ‘jumps’ on a number line e.g. to add 142 to a number they may ‘jump’ 100 and then 40 and then 2. The number line is sometimes used in multiplication and division but can be time consuming.

Number track – a sequence of numbers, each inside its own square. It is a simplified version of the number line that emphasises the whole numbers.

Numicon – practical maths equipment that teaches children the names and values of numbers 1-10 initially but then helps them with early addition, subtraction, multiplication and division. Numicon is useful for showing the real value of a number practically.

One-Step Calculation – a calculation involving only one operation e.g. addition. Usually the child must decide what that operation is.

Partition – split up a larger number into parts, such as the hundreds, tens and units e.g. 342 can be partitioned into 300 and 40 and 2

Place Value – the value of a digit created by its position in a number e.g. 3 represents thirty in 234 but three thousand in 3567

Recombine – for addition, once you have partitioned numbers into hundreds, tens and units then you have to add the hundreds together, then add the tens to that total, then add the units to that total

Remainder – a whole number left over after a division calculation

Repeated addition – repeatedly adding groups of the same size for multiplication

Scaling – an approach to multiplication whereby the number is ‘scaled up’ by a factor of the multiplier e.g. 4×3 means 4 scaled up by a factor of 3.

Sharing – an approach to division whereby the dividend is shared out into a given number of groups (like dealing cards)

Short Division - traditional method for division with a single digit divisor (this is a compact version of long division, sometimes called ‘bus stop’)

Significant digit – the digit in a number with the largest value e.g. in 34 the most significant digit is the 3, as it has a value of ‘30’ and the ‘4’ only has a value of ‘4’

Single digit – a number with only one digit. These are always less than 10.

Sum – the total of two or more numbers (it implies addition). Sum should not be used as a synonym for calculation.

Two-step calculation - a calculation where two different operations must be applied e.g. to find change in a shop you will usually have to add the individual prices and then subtract from the total amount. Usually the child has to decide what these two operations are and the order in which they should be applied.