Abingdon Primary School

Calculation Policy

January 2020

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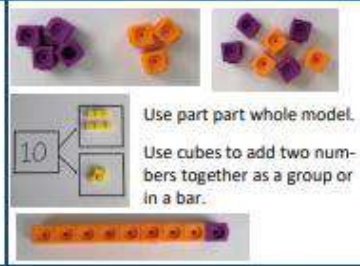
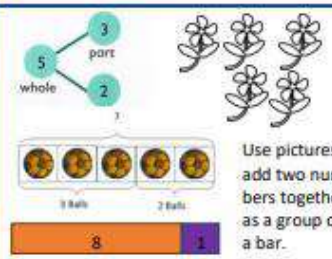
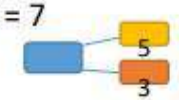
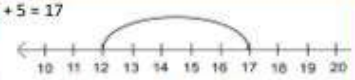
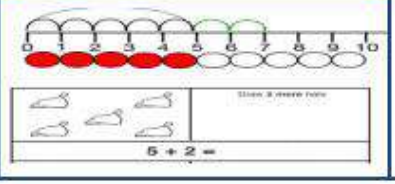
Calculation Policy

Principles

- This calculation policy is focused on developing proficiency with the expected formal written methods by the end of Year 6 and hence the progression guidance provided for each operation is designed to flow into the expected method as exemplified on the National Curriculum Appendix document.
- Specific practical equipment and approaches have been suggested for each age group to support children in developing the conceptual understanding that will enable them to move more rapidly and efficiently towards the formal written methods expected.
- It is recommended that teachers encourage children to simultaneously carry out the calculation practically using the equipment/representation suggested and to record this calculation step by step using the parallel formal written method.
- It is expected that children will work towards the fluency goals for each age group but that, where necessary, teachers will use approaches and materials from earlier year groups to bridge any gaps in a child's understanding.
- Teachers should have an understanding of the expectations and progression for all year groups, regardless of which year group they teach.
- The 'Written Methods', 'With jottings ...or in your head' and 'Just know it' sections list the national curriculum expectations of the year group for calculation.
- The 'Developing Conceptual Understanding' section illustrates how to build children's understanding of the formal methods using a range of specific practical equipment and representations. The expected language for the formal methods is modelled in this section in the older year groups – this language should be used throughout whenever the formal method is used.
- The 'Foundations' section for each year group highlights the skills and knowledge that should be addressed on a regular basis within this year group to ensure that children have the requisite fluency to address the new approaches required.
- **When modelling word problems, use part whole and bar model to give children an image they can use and transfer to all other problems.**
- **When calculating with fractions, use the bar model to support.**

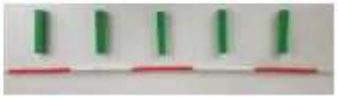
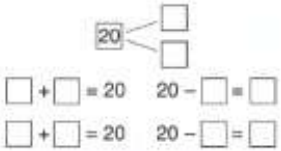
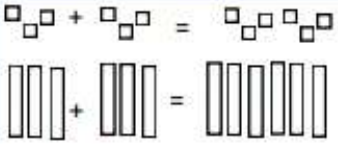
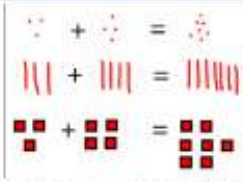
Key Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line , equal to, how many more, most, least, count back

Y1 ADDITION +

Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	 Use part part whole model. Use cubes to add two numbers together as a group or in a bar.	 Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7  10= 6 + 4 Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. *This is an essential skill for column addition later.*	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10. Use ten frames.	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Represent & use number bonds and related subtraction facts within 20	 2 more than 5.	 6 + 2 =	Emphasis should be on the language *'1 more than 5 is equal to 6.'* *'2 more than 5 is 7.'* *'8 is 3 more than 5.'*

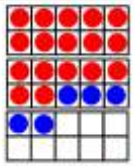
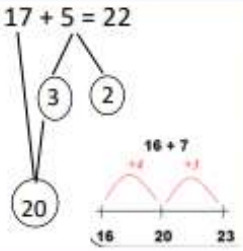
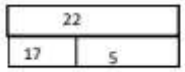
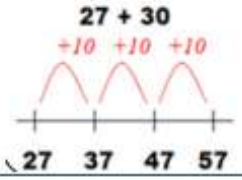
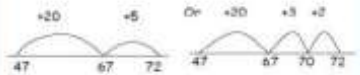
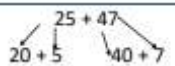
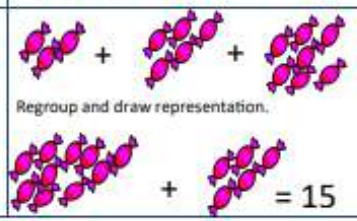
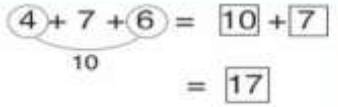
Key Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line , equal to, how many more, most, least, count back, count on, strategy, sum, tens, units, ones, partition, addition, column, tens boundary

Y2 ADDITION +

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	50= 30 = 20  Model using dienes and bead strings	 Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Use known number facts Part part whole	 Children explore ways of making numbers within 20	 □ + □ = 20 20 − □ = □ □ + □ = 20 20 − □ = □	□ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1
Using known facts		 Children draw representations of H,T and O	3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700
Bar model	 3 + 4 = 7	 7 + 3 = 10	 23 + 25 = 48

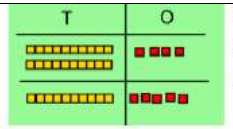
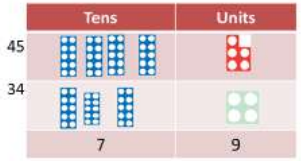
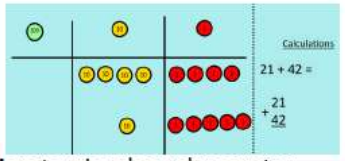
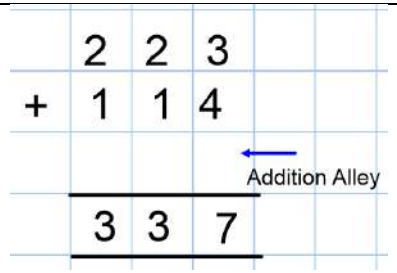
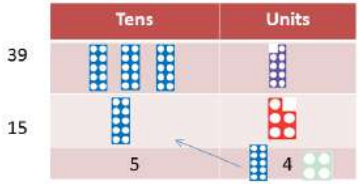
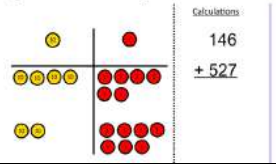
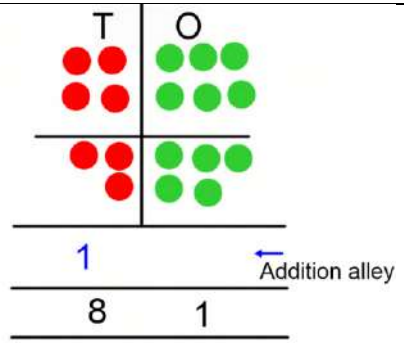
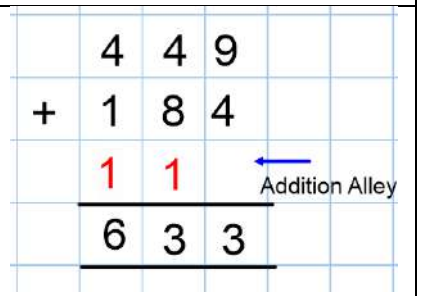
Key Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line , equal to, how many more, most, least, count back, count on, strategy, sum, tens, units, ones, partition, addition, column, tens boundary

Y2 ADDITION +

Objective & Strategy	Concrete	Pictorial	Abstract
Add a two digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	Use part part whole and number line to model. 	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 
Add a 2 digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change		27 + 10 = 37 27 + 20 = 47 27 + □ = 57
Add two 2-digit numbers	 Model using dienes , place value counters and numicon	 Use number line and bridge ten using part whole if necessary.	 20 + 40 = 60 5+ 7 =12 60 + 12 = 72
Add three 1-digit numbers	 Combine to make 10 first if possible, or bridge 1o then add third digit	 Regroup and draw representation.	 Combine the two numbers that make/ bridge ten then add on the third.

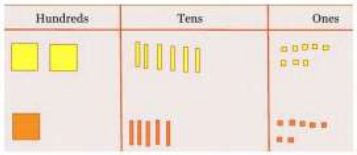
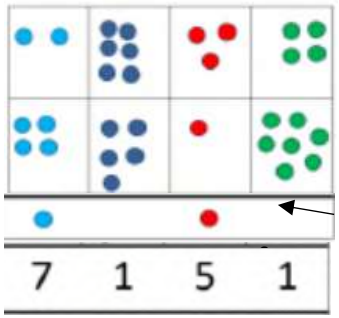
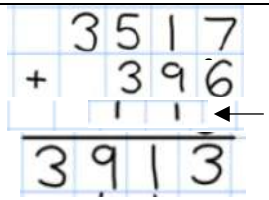
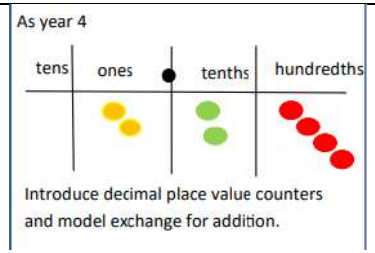
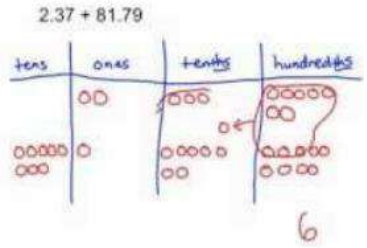
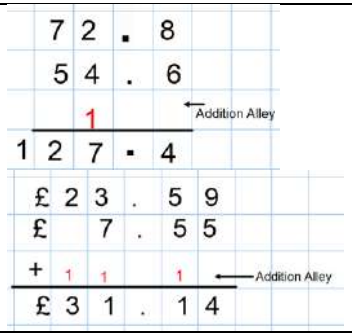
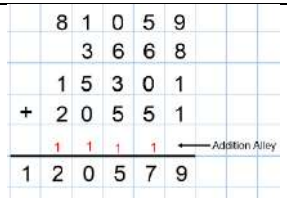
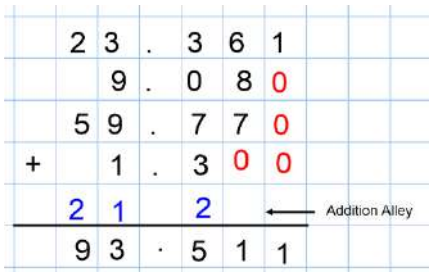
Key Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line , equal to, how many more, most, least, count back, count on, strategy, sum, tens, units, ones, partition, addition, column, tens boundary exchange, hundreds, value, digit, hundreds boundary, increase, vertical, 'carry', expanded, compact

Y3 ADDITION +

Objective & Strategy	Concrete	Pictorial	Abstract
Children should be taught column addition with no carrying before moving to number crossing the boundary. Column Addition—no regrouping (friendly numbers) Add two or three 2 or 3-digit numbers	 Model using Dienes or numicon Add together the ones first, then the tens.   Move to using place value counters	Children move to drawing the counters using a tens and one frame.  Children draw Addition Alley even though it is not needed yet to understand what is to come.	 Add the ones first, then the tens, then the hundreds.
Column Addition with 'carrying' crossing the tens and hundreds barrier.	 Exchange ten ones for a ten. Model using numicon and pv counters. 		

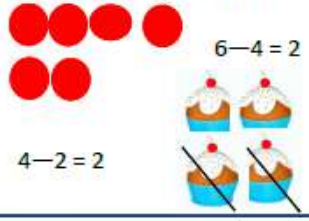
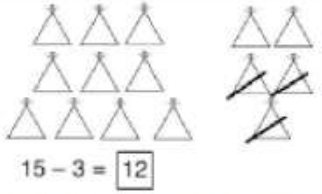
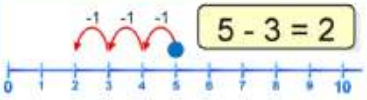
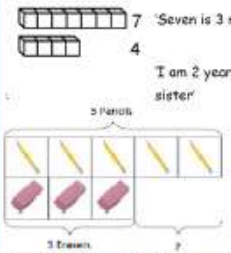
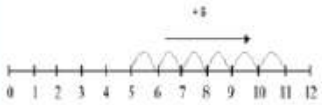
Key Vocabulary: See previous year groups vocabulary also. exchange, hundreds, value, digit, hundreds boundary, increase, vertical, 'carry', expanded, compact, inverse thousands, hundreds, digits, inverse Yr4 (tenths, hundredths, thousandths, decimal point, decimal, decimal places Yr5/6)

Y4-6 Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Y4—add numbers with up to 4 digits **When addition alley has been introduced, it will continue to be used when using formal written method.**	Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 Draw representations using pv grid.	 Addition alley Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.			
Y6—add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	As Y5		

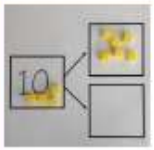
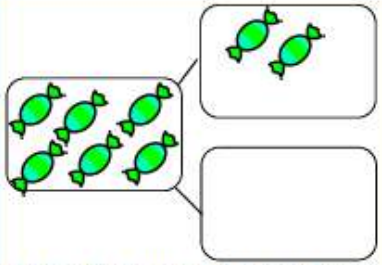
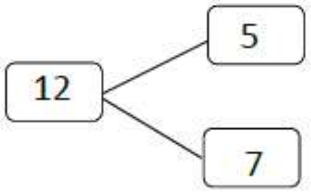
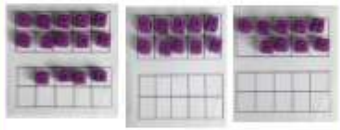
Key Vocabulary: total, equal to, equals, count on, number line , equal to, take, take away, less, minus, subtract, leaves, distance between, how many fewer / less than, most, least, count back , how many left, how much less is_?

Y1 SUBTRACTION

Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters , cubes etc to show how objects can be taken away. 	 Cross out drawn objects to show what has been taken away.	7—4 = 3 16—9 = 7
Counting back	 Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	 Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Find the Difference	Compare objects and amounts  Lay objects to represent bar model.	Count on using a number line to find the difference. 	Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister.?

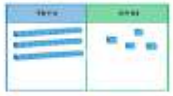
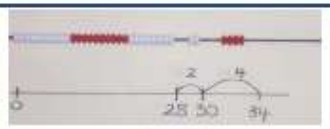
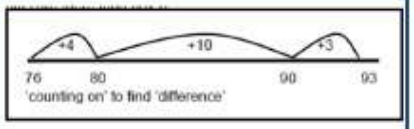
Key Vocabulary: total, equal to, equals, count on, number line , equal to, take, take away, less, minus, subtract, leaves, distance between, how many fewer / less than, most, least, count back , how many left, how much less is_?

Y1 SUBTRACTION -

Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what s the other part? 10—6 = 4	 Use pictorial representations to show the part.	Move to using numbers within the part whole model. 
Make 10	14—9  Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	13—7  Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off?
Bar model	 5—2 = 3		8 | 2 10 = 8 + 2 10 = 2 + 8 10—2 = 8 10—8 = 2

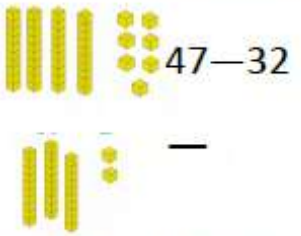
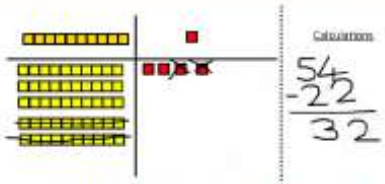
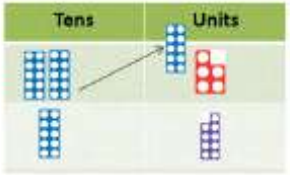
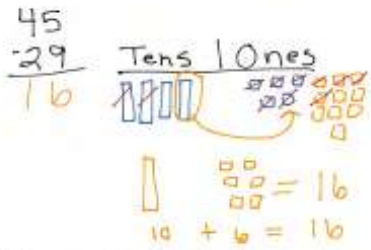
Key Vocabulary: total, equal to, equals, count on, number line , equal to, take, take away, less, minus, subtract, leaves, distance between, how many fewer / less than, most, least, count back , how many left, how much less is_?difference, count on, strategy, sum, tens, units, ones, partition, column, tens boundary

Y2 SUBTRACTION -

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'		20—4 = 16
Partitioning to subtract without regrouping. *'Friendly numbers'*	34—13 = 21    Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.  43—21 = 22	43—21 = 22
Make ten strategy *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.*	 34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to next ten and then the rest.	93—76 = 17

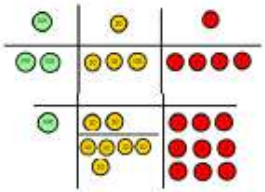
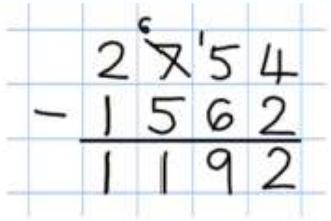
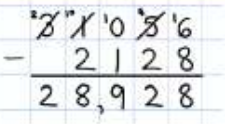
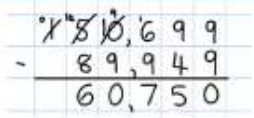
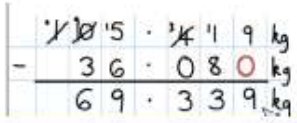
Key Vocabulary: total, equal to, equals, count on, number line , equal to, take, take away, less, minus, subtract, leaves, distance between, how many fewer / less than, most, least, count back , how many left, how much less is_?**difference, count on, strategy, sum, tens, units, ones, partition, column, tens boundary** exchange, decrease, hundreds, value, digit, hundreds boundary, increase, vertical, 'carry', expanded, compact

Y3 SUBTRACTION -

Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	 Use base 10 or Numicon to model	 Darw representations to support understanding	$47-24=23$ $40+7$ $-\ 20+4$ $20+3$ Intermediate step may be needed to lead to clear subtraction understanding. 
Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into tten ones. Use the phrase 'take and make' for exchange.	 Children may draw base ten or PV counters and cross off.	 Begin by partitioning into pv columns  Then move to formal method.

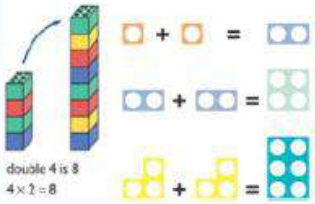
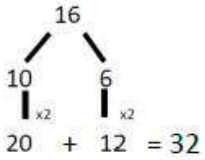
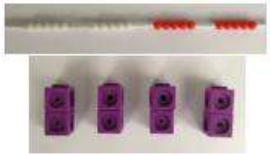
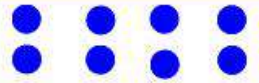
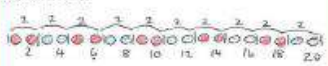
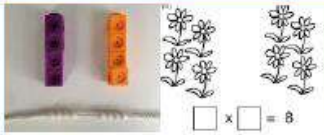
Key Vocabulary See previous year groups vocabulary also inverse thousands, hundreds, digits, inverse Yr4
tenths, hundredths, thousandths, decimal point, decimal, decimal places Yr5/6

Y4-6 SUBTRACTION -

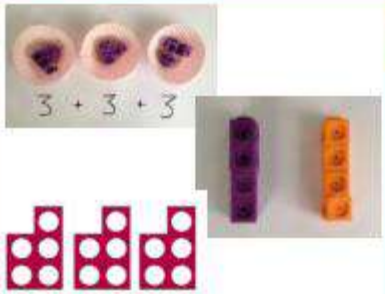
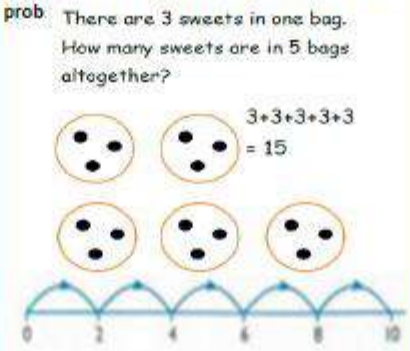
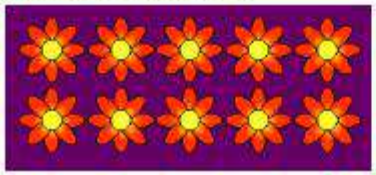
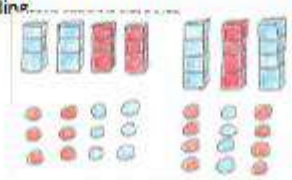
Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179  Model process of exchange using Numicon, base ten and then move to PV counters.	Children to draw pv counters and show their exchange—see Y3	 Use the phrase 'take and make' for exchange
Year 5- Subtract with at least 4 digits, including money and measures. *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal*	As Year 4	Children to draw pv counters and show their exchange—see Y3	 Use zeros for place-holders. 
Year 6—Subtract with increasingly large and more complex numbers and decimal values.			 

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count

Y1 MULTIPLICATION X

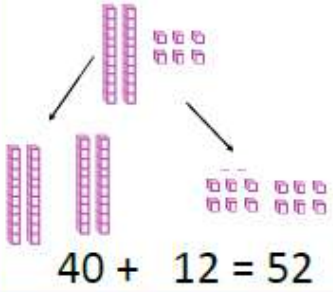
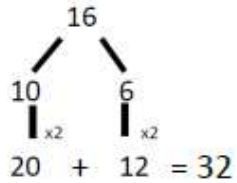
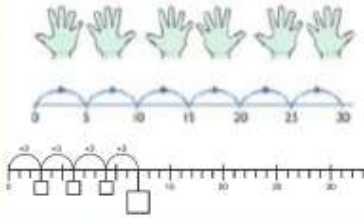
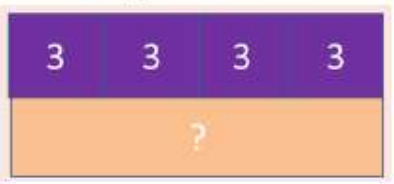
Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling 	Draw pictures to show how to double numbers 	Partition a number and then double each part before recombining it back together. 
Counting in multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting. 	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Making equal groups and counting the total	 Use manipulatives to create equal groups.	Draw  to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8

Y1 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	 Use different objects to add equal groups	Use pictorial including number lines to solve prob 	Write addition sentences to describe objects and pictures. 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding 	3 x 2 = 6 2 x 5 = 10

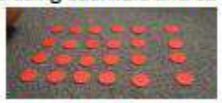
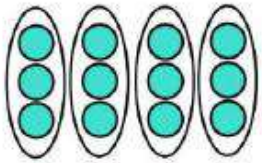
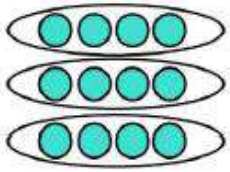
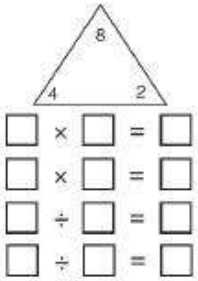
Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times. number line

Y2 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters.  40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together.  16 10 (x2) 6 (x2) 20 + 12 = 32
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40   ?	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.   3 3 3 3 ?	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 4 × 3 = □

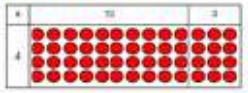
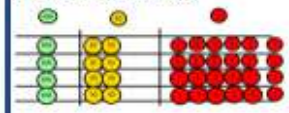
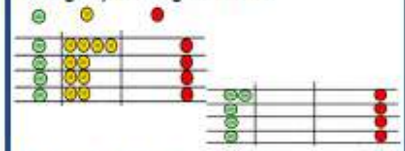
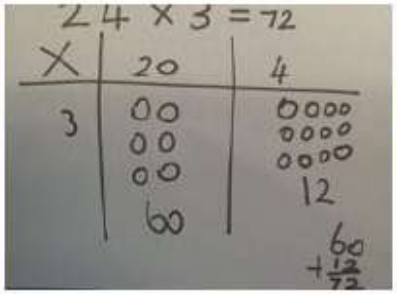
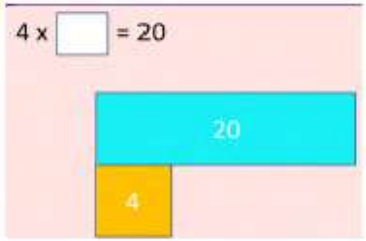
Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times. number line

Y2 MULTIPLICATION

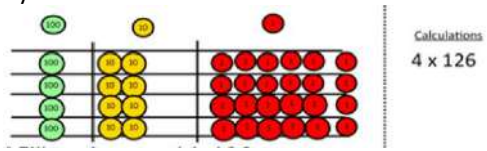
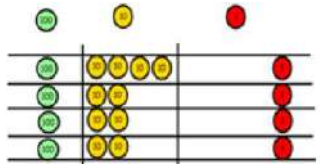
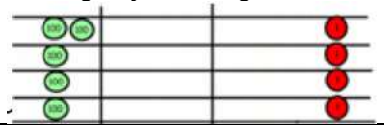
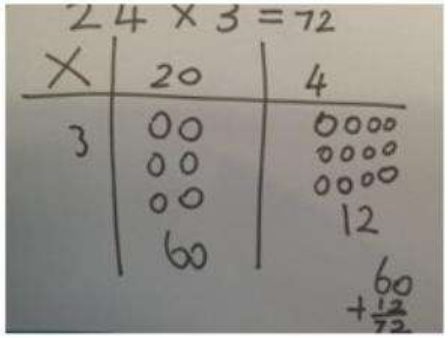
Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	12 = 3 × 4 12 = 4 × 3 Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times. number line partition, grid method, multiple, product, tens, units, value, inverse, multiple

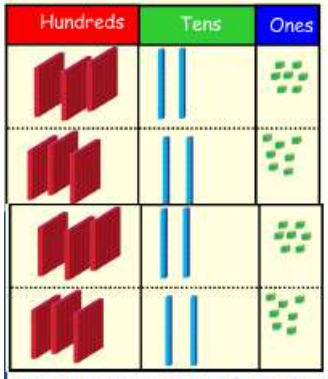
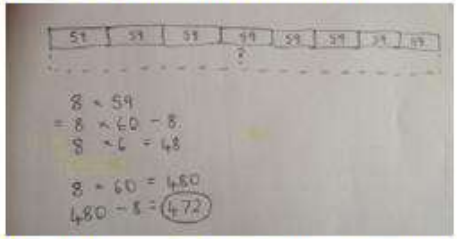
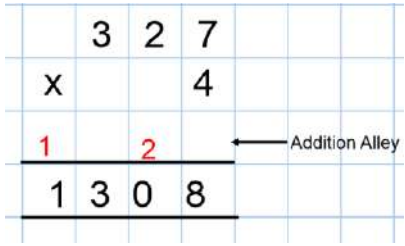
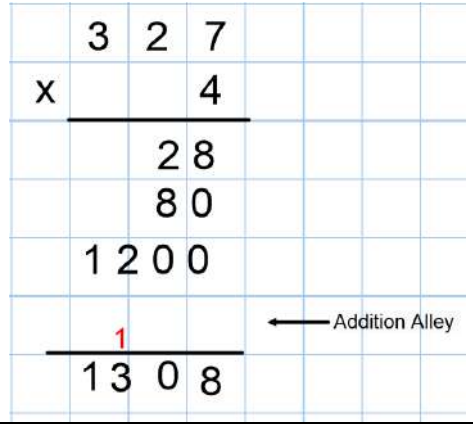
Y3 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method	Show the links with arrays to first introduce the grid method  4 rows of 10 4 rows of 3 Move onto base ten to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  4 x 126 Fill each row with 126  4 x 126 Add up each column, starting with the ones making any exchanges needed  Then you have your answer.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  Bar model are used to explore missing numbers  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. × | 30 | 5 7 | 210 | 35 210 + 35 = 245 Moving forward, multiply by a 2 digit number showing the different rows within the grid method. 10 | 8 10 | 100 | 80 3 | 30 | 24

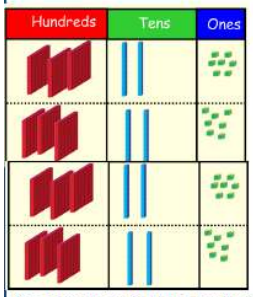
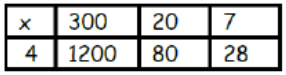
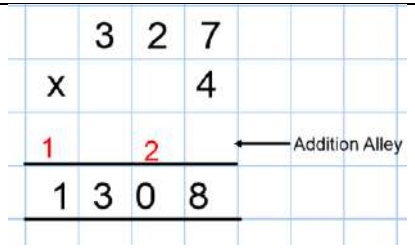
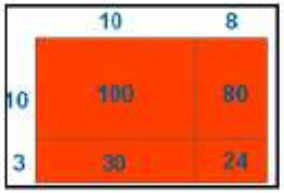
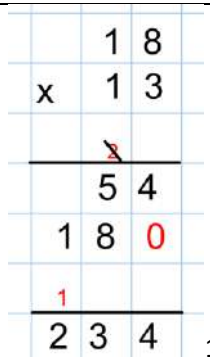
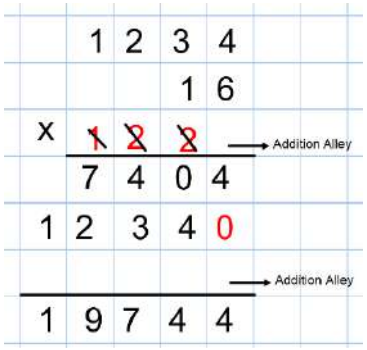
Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times. number line partition, grid method, multiple, product, tens, units, value, inverse, multiple **inverse**

Objective & strategy	Concrete	Pictorial	Abstract
Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Fill each row with 126  Add up each column, starting with the ones making any exchanges needed 	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | x | 30 | 5 | | --- | --- | --- | | 7 | 210 | 35 | 210 + 35 = 245

Y4 MULTIPLICATION

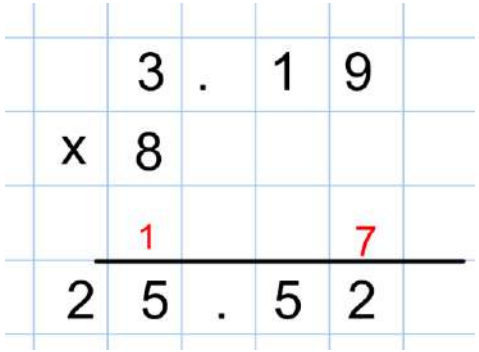
Column Multiplication	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside	<table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table> The grid method my be used to show how this relates to a formal written method.  Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	 Compact method If children can not at first move to the compact method, they may need to use expanded first. See below.  	

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times. number line partition, grid method, multiple, product, tens, units, value, inverse, multiple square, factor, integer, decimal, short/long multiplication, 'carry', quotient, prime number, prime factors, composite number (non-prime)

Objective & strategy	Concrete	Pictorial	Abstract
Column Multiplication for 3 and 4 digits x 1 digit.	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside	 	
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	 Continue to use bar modelling to support problem solving	 Needing two addition alleys. The first one, the numbers are crossed off 

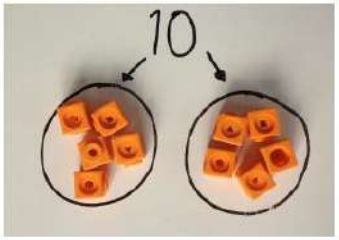
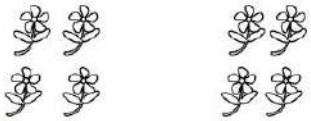
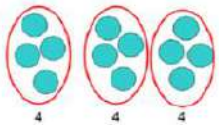
16

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times. number line partition, grid method, multiple, product, tens, units, value, inverse, multiple tenths, hundredths, decimal, common factor, common multiple

Objective & strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 

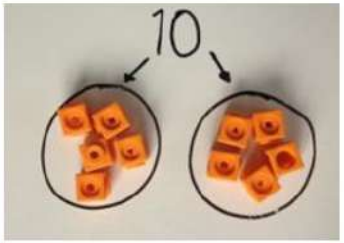
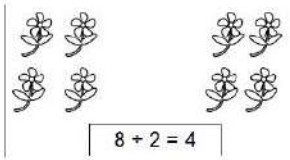
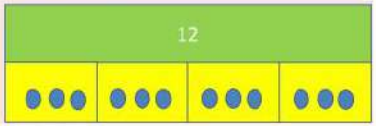
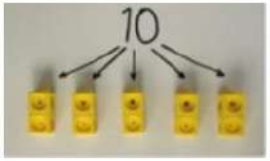
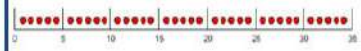
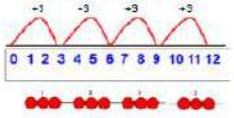
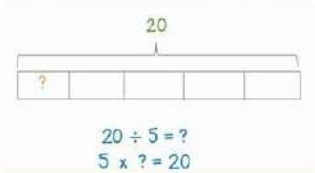
Y6 MULTIPLICATION X

Key Vocabulary: groups of, lots of, array, altogether, multiply, count, share, share equally, one each, two each…, group, groups of, lots of

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing *Use Gordon ITPs for modelling*	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 shared between 2 is 4 Sharing:  12 shared between 3 is 4	12 shared between 3 is 4

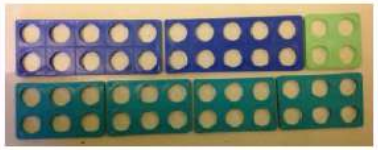
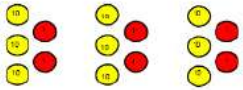
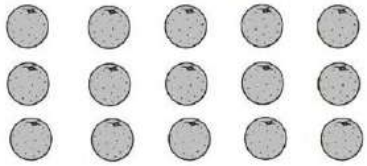
Y1 DIVISION ÷

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, **share, share equally, one each, two each…, group, equal groups of,** multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times… **divide, divided by, divided into, division, grouping, number line, left, left over**

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	12 ÷ 3 = 4
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  12 ÷ 3 = 4 Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  20 ÷ 5 = ? 5 x ? = 20	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

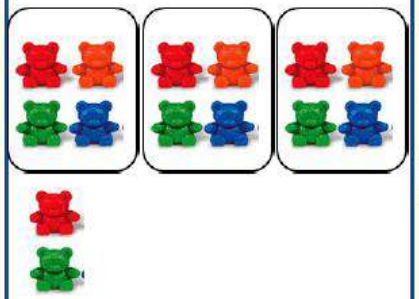
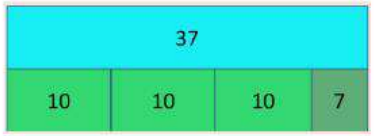
Y2 DIVISION ÷

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times..., share, share equally, one each, two each..., group, equal groups of, divide, divided by, divided into, division, grouping, number line, left, left over, partition, grid method, multiple, product, tens, units, value, inverse, short division, 'carry', remainder, multiple

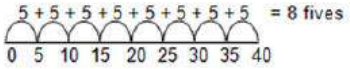
Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 96 ÷ 3 = 32 	Continue to use bar modelling to aid solving division problems.  20 ÷ 5 = ? 5 x ? = 20	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

Y3 DIVISION ÷

Y3 DIVISION ÷

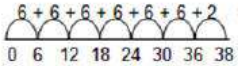
Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  	Complete written divisions and show the remainder using r. 

Example without remainder:
40 ÷ 5
Ask "How many 5s in 40?"



= 8 fives

Example with remainder:
38 ÷ 6



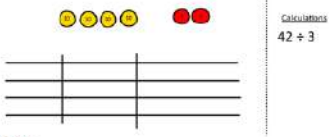
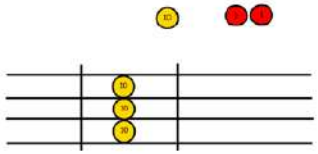
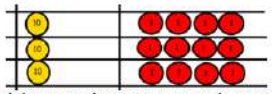
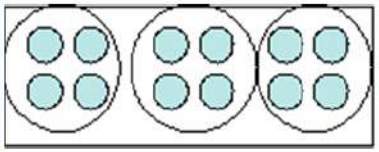
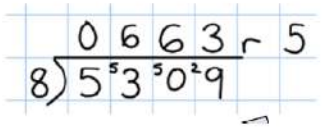
= 6 sixes with a remainder of 2

For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of, share, share equally, one each, two each…, group, equal groups of, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, "carry", remainder, multiple, inverse, divisible by, factor

Y4-6

DIVISION ÷

Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	96 ÷ 3 Tens Units 3 2 3  Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. 2 1 8 3 4 8 7 2 Move onto divisions with a remainder. 8 6 r 2 3 5 4 3 2 Finally move into decimal places to divide the total accurately. 1 4 . 6 16 21 3 5 5 1 1 . 0 

Long Division

Step 1—a remainder in the ones

```
  h t o
  0 4 1 R1
4 ) 1 6 5
```

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

```
   th h t o
   0 4 0 0 R7
8 ) 3 2 0 7
```

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

```
    h t o
    0 6 1
4 ) 2 4 7
    - 4
      3
```

When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3.

Check: 4 × 61 + 3 = 247

```
   th h t o
   0 4 0 2
4 ) 1 6 0 9
     - 8
       1
```

When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

Y6

DIVISION ÷

Long Division

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2)5 8	t o 2 2)5 8 -4 1	t o 2 9 2)5 8 -4 ↓ 1 8
Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2)5 8 -4 1 8	t o 2 9 2)5 8 -4 1 8 -1 8 0	t o 2 9 2)5 8 -4 1 8 -1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

Y6

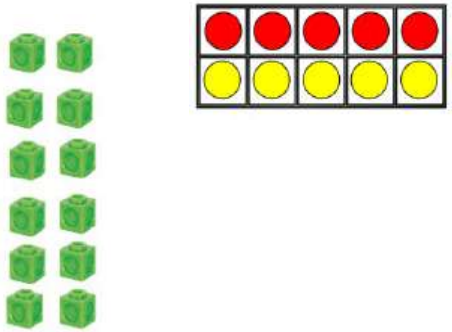
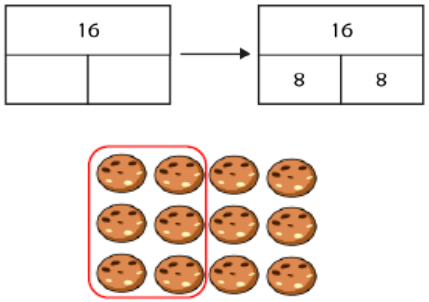
DIVISION ÷

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)2 7 8	h t o 1 2)2 7 8 - 2 0	h t o 1 8 2)2 7 8 - 2 ↓ 0 7
Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 2)2 7 8 - 2 0 7	h t o 1 3 2)2 7 8 - 2 0 7 - 6 1	h t o 1 3 2)2 7 8 - 2 0 7 - 6 1 8
Divide 2 into 7. Place 3 into the quotient.	Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8	h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8 - 1 8 0	h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	There are no more digits to drop down. The quotient is 139.

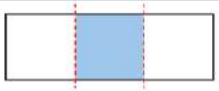
Y6 DIVISION ÷

Key Vocabulary: whole, parts of a whole fraction, one whole, half, quarter, three-quarters

Concrete	Pictorial	Abstract
Find ½ of __ Find half of ___ 	Find ½ of 16 Find half of 16 	Find ½ of 16 Find half of 16 $\frac{1}{2}$ of 4 = ☐ $\frac{1}{2}$ of 6 = ☐ $\frac{1}{2}$ of 8 = ☐

Model	Say	Write	Notation
 one-half	*'The apple has been divided…'*	Write the division bar.	$\frac{1}{2}$
	'…into 2 equal parts…'	Write '2' as the denominator.	
	'…and we have 1 of the parts.'	Write '1' as the numerator.	

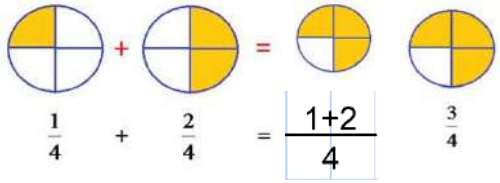
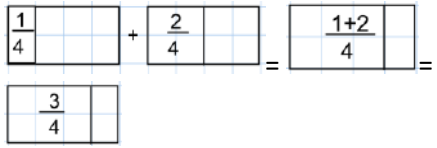
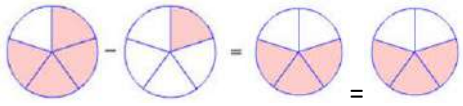
A key concept is for teachers to use the vocabulary in Key Stage 1 of 'part' and 'whole' and to always refer back to this vocabulary ready for Key Stage 2.

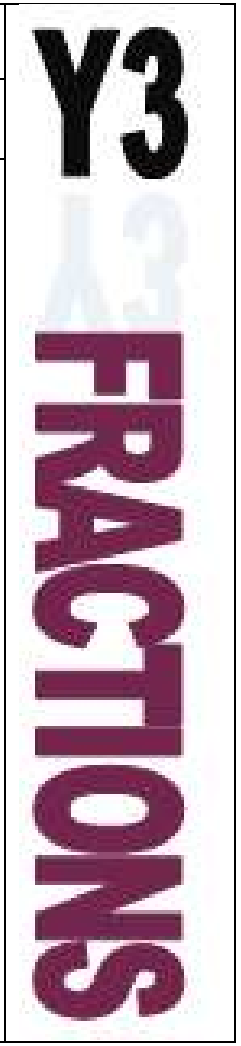
Model	Say	Write	Notation
 one-third	*'The rectangle has been divided…'*	Write the division bar.	$\frac{1}{3}$
	'…into 3 equal parts…'	Write '3' as the denominator.	
	'…and 1 of the parts is shaded.'	Write '1' as the numerator.	

Model	Say	Write	Notation
 one-quarter	*'The strawberries have been divided…'*	Write the division bar.	$\frac{1}{4}$
	'…into 4 equal parts…'	Write '4' as the denominator.	
	'…and 1 of the parts is circled.'	Write '1' as the numerator.	

Y2 FRACTIONS

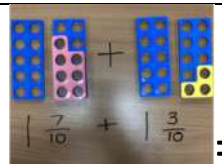
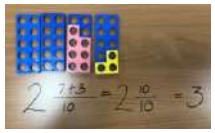
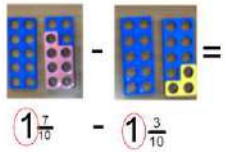
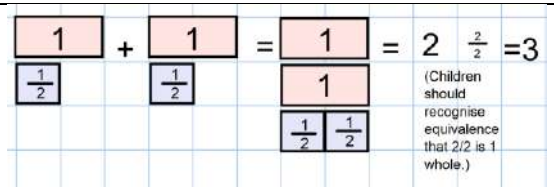
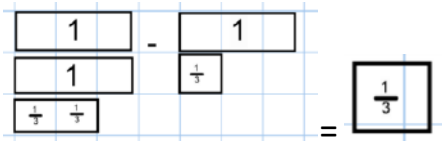
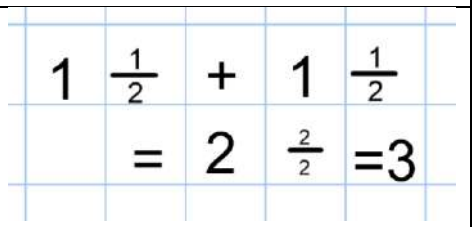
Key Vocabulary: fraction, one whole, half, quarter, three-quarters, add, subtract, equal parts of a whole, numerator, denominator

Objective & strategy	Concrete	Pictorial	Abstract
Add and subtract fractions with the same denominator within one whole (e.g. $^{5}/_{7} + {}^{1}/_{7} = {}^{6}/_{7}$)	 Introduce stem sentence ' **when the denominators are the same, normal rules of arithmetic apply.'**  $\frac{5}{10} + \frac{3}{10} = \frac{5+3}{10} \quad \frac{8}{10}$ **Subtraction**  Use a variety of resources such as number rods, paper strips, equivalence circles, cards to model what happens when you add and subtract fractions with the same denominator.	 $\frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} \quad \frac{3}{4}$ Bar model  **Subtraction**  $\frac{4}{5} - \frac{1}{5} = \frac{4-1}{5} = \frac{3}{5}$	$\frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} = \frac{3}{4}$ **Subtraction** $\frac{4}{5} - \frac{1}{5} = \frac{4-1}{5} = \frac{3}{5}$

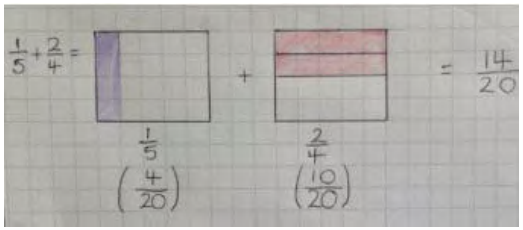
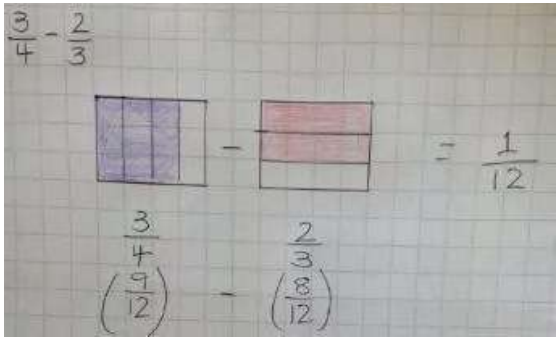
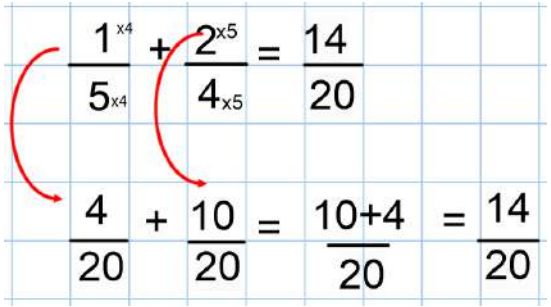
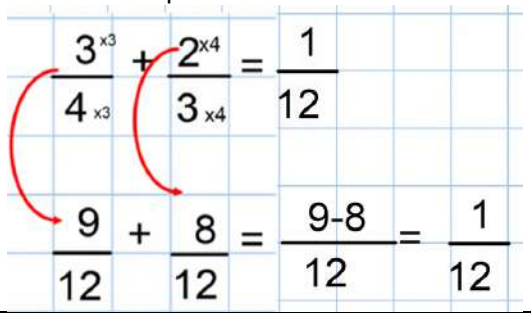


Key Vocabulary: fraction, whole-one/s, half, quarter, three-quarters, numerator, denominator, add, subtract, equal parts of a whole, mixed number, equivalence, equivalent

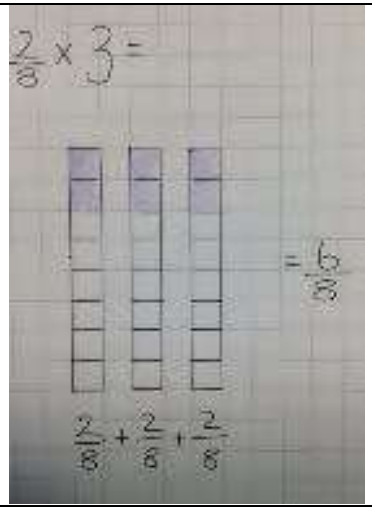
Y4 FRACTIONS

Objective & strategy	Concrete	Pictorial	Abstract
Add and subtract fractions with the same denominator Continue to develop addition and subtraction of fractions as shown in Year 3 but beyond the whole one, using lots of practical resources such as number rods, equivalence circles, cards, etc to ensure conceptual understanding.	  **Children should understand the vocabulary parts and wholes. Children should deal with the wholes first then the parts.** **Subtractions**  Children deal with the wholes first then the parts. Only exchanging if the fraction goes below a whole.	 Bar model Children will add the whole ones first. **1whole +1 whole = 2 wholes.** Then, add the parts ½ +1/2 = 1 whole. Then use equivalence to recognise that 2/2 = 1 whole. The same concept for subtraction calculations is to be followed. **Subtractions** 	 **Subtractions** 

Key Vocabulary: fraction, whole-one/s, half, quarter, three-quarters, numerator, denominator, add, subtract, equal parts of a whole, mixed number, equivalence, equivalent, array, model, improper fraction, multiples, common denominator

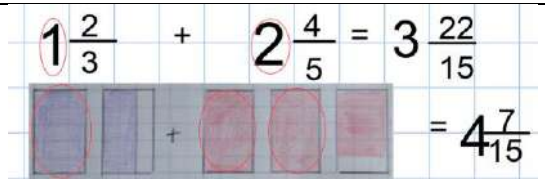
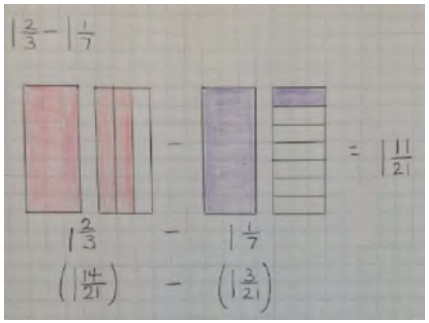
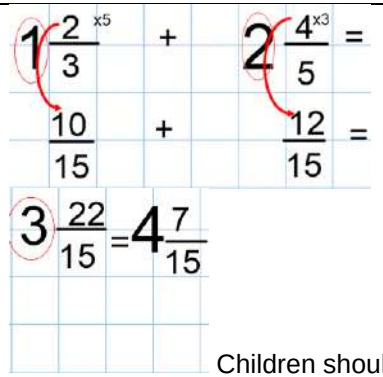
Objective & strategy	Concrete	Pictorial	Abstract
Add and subtract fractions with the same denominator and multiples of the same number		Addition  The grid is drawn 5x4 for both to show the common multiple. Subtraction 	Children use common multiples to find the common denominator.  Children should try and simplify at the end of the calculation if possible. 

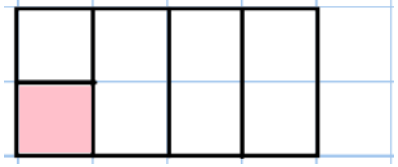
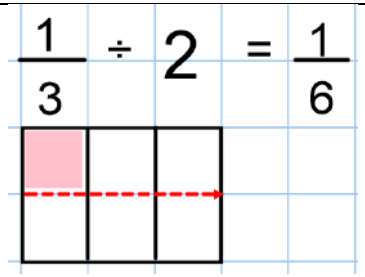
Y5 FRACTIONS

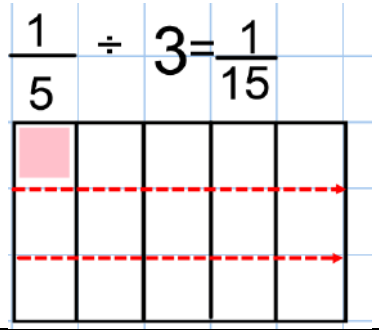
Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams			A similar model can be used for repeated addition for multiplication of fractions and mixed numbers by whole numbers	$\frac{3}{6} \times 7 = \frac{3x7}{6} = \frac{21}{6}$ Children should then simplify into a mixed number.	

Key Vocabulary: : fraction, whole-one/s, half, quarter, three-quarters, numerator, denominator, equal parts of a whole, mixed number, equivalence, equivalent, array, model, improper fraction, common denominator, multiples, multiply, repeated addition

Y6 Spring FRACTIONS

Objective & strategy	Concrete	Pictorial	Abstract
Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions Children should continually be taught to deal with the wholes first then the parts.		 Children should continually be taught to deal with the wholes first then the parts. Children simplify at the end . **Subtraction**  When children are able to move on from the above model, they might encounter difficulties when using mixed numbers. For example, if it was 1 ¼ - ½ this would require the understanding that they would need to break the whole one down into quarters. To ensure that children do not become confused by this, they should be taught to convert the mixed numbers to improper fractions first and then converted back, once the calculation is done.	 Children should continually be taught to deal with the wholes first then the parts.

Multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g. $^1/_4 \times {}^1/_2 = {}^1/_8$)		$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$  Step 1: draw a grid that shows both quarters and halves At this point, only draw lines that show quarters of the grid because this is what is being multiplied. Step 2: Look at one of the quarters and shade half of it (because we are multiplying by a half). At this point children will need to know that when multiplying by a fraction, the number does not get bigger, it actually gets smaller. Step 3: Find the fraction that is shaded by drawing an imaginary line across the whole grid halving it also. How many equal parts are shaded? (Numerator) How many equal parts has the grid been divided into? (Denominator)	$\frac{6}{10} \times \frac{3}{4} = \frac{6x3}{10x4} = \frac{18}{40}$ Children should see from the pictorial explanation that the numerators are multiplied together, and the denominators are multiplied together.	
Divide proper fractions by whole numbers (e.g. $^1/_3 \div 2 = {}^1/_6$)		$\frac{1}{3} \div 2 = \frac{1}{6}$  Step 1: draw a grid that shows the dividend (1/3 in this case) and also the divisor (2 in this case).	$\frac{1}{3} \div 2 = \frac{1}{3x2} = \frac{1}{6}$ $\frac{1}{5} \div 3 = \frac{1}{5x3} = \frac{1}{15}$	

		Draw lines down to show the dividend (1/3) but do not draw lines across to show the divisor at this point. Step 2: look at one of the thirds and divide it by 2 (the divisor). Shade in one part of this. Step 3: To find the answer, look at the fraction that is shaded by drawing an imaginary line across the whole grid, dividing it by 2. How many equal parts are shaded? (Numerator) How many equal parts has the grid been divided into? (Denominator). In this case the answer is 1/6 		

Year 1

Addition and Subtraction

Video Clip: https://www.youtube.com/watch?v=OkW1Y11tGxw&list=UUVb98bWNgEmk02R7enUrmFA

Year 2

Addition and Subtraction

Video: https://www.youtube.com/watch?v=mEHKmMapWGY

Multiplication and Division

Multiple Representations of Multiplication
https://www.youtube.com/watch?v=YPWmOVt8vgw&list=UUVb98bWNgEmk02R7enUrmFA

The Commutative Law for Multiplication
https://www.youtube.com/watch?v=VGkjjVfnGYI&list=UUVb98bWNgEmk02R7enUrmFA

Sharing and Grouping (whole class)
http://vimeo.com/83485518

Sharing and Grouping (pairs)
http://vimeo.com/83485658

Year 3

Addition and Subtraction

Videos:

Subtraction—teaching children to consider the most appropriate methods before calculating http://www.youtube.com/watch?v=RCCLseBLBSo

Introducing partitioned column subtraction method, from practical to written
http://www.youtube.com/watch?v=dP8NlFLZzOg

Multiplication and Division

Videos: (Y4 examples but can be adapted to suit Year 3)
Representing division with place value counters **http://vimeo.com/83485661**
Using place value counters and recording division http://vimeo.com/83485662

Year 4

Addition and Subtraction

Subtraction—teaching children to consider the most appropriate methods before calculating http://www.youtube.com/watch?v=RCCLseBLBSo

Introducing partitioned column subtraction method, from practical to written
http://www.youtube.com/watch?v=dP8NlFLZzOg

Moving to the compact column method of subtraction http://www.youtube.com/watch?v=3ihxp2mqnhs

Multiplication and Division

Multiplication – Lower Key Stage 2 (links to place value counters as shown in Y3) http://vimeo.com/70319240

Representing division with place value counters **http://vimeo.com/83485661**

Using place value counters and recording division http://vimeo.com/83485662

Year 5

Addition and Subtraction

Video:
Moving to the compact column method of subtraction http://www.youtube.com/watch?v=3ihxp2mqnhs

Multiplication and Division

Multiplication – Upper Key Stage 2 http://vimeo.com/70318365

Rapid Recall of Multiplication Facts https://www.youtube.com/watch?v=BcIjRLZzMaw&list=PLQqF8sn28L9wjDm8uJEJcRCDDoY6raPE_&index=2
Year 6
Addition and Subtraction See videos in Years 4 and 5 to show understanding of compact method
Multiplication and Division **Videos**: Rapid Recall of Multiplication Facts https://www.youtube.com/watch?v=BcIjRLZzMaw&list=PLQqF8sn28L9wjDm8uJEJcRCDDoY6raPE_&index=2

Glossary of Terms

2-digit number– a number with 2 digits like 23, 45, 12 or 60
3-digit number – a number with 3 digits like 123, 542, 903 or 561
Addition facts – knowing that 1+1 = 2 and 1+3 = 4 and 2+5 = 7. Normally we only talk about number facts with totals of 20 and under.
Array - An array is an arrangement of a set of numbers or objects in rows and columns –it is mostly used to show how you can group objects for repeated addition or subtraction.
Bead String/Bar – a string with (usually 100) beads on, grouped by colour in tens. The bead string is a good bridge between a number track and a number line as it maintains the cardinality of the numbers whilst beginning to develop the concepts of counting 'spaces' rather than objects.
Bridging – when a calculation causes you to cross a 'ten boundary' or a 'hundred boundary' e.g. 85 + 18 will bridge 100.
Compact vertical – the name of the recommended written method for addition whereby the numbers are added in columns, 1s first then 10s and so on. Where the total exceeds 10, the ten 1s are exchanged for a 10 and written below the answer line. Sometimes referred to as 'carrying'.
Concrete apparatus – objects to help children count and calculate– these are most often cubes (multilink) but can be anything they can hold and move including Cuisenaire rods, Dienes rods (hundreds, tens and units blocks), straws, Numicon, Place Value counters and much more.
Count all – when you add by counting all the items/objects e.g. to add 11 and 5 you would count out 11, then count out 5, then put them together and count them all to get 16.

Count on – when you add (or sometimes subtract) by counting onwards from a given number. E.g. to add 11 and 5 you would count on 5 from 11 i.e. 12, 13, 14, 15, 16

Decimal number – a number with a decimal point e.g. 2.34 (said as two point three four)

Decomposition – the name of the recommended written method for subtraction whereby the smaller number is subtracted from the larger, 1s first then 10s and so on. Where the subtraction cannot be completed as the second number is larger than the first, a 10 is exchanged for ten 1s to facilitate this. This is the traditional 'borrowing' form of column method, which is different to the 'payback' method.

Dienes Rods (or Base 10) – this is a set of practical equipment that represents the numbers to help children with place value and calculation. The Dienes rods show 1s, 10s, 100s and 1000s as blocks of cubes that children can then combine. Dienes rods do not break up so the child has to 'exchange' them for smaller or larger blocks where necessary.

Difference – the gap between numbers that is found by subtraction e.g. 7-5 can be read as '7 take away 5' or as the 'difference between 7 and 5'

Dividend – the number being divided in a calculation

Divisor – the smaller number in a division calculation.

Double – multiply a number by 2

Efficient Methods – the method(s) that will solve the calculation most rapidly and easily

Equals - is worth the same as (be careful not to emphasise the use of = to show the answer)

Exchanging – Swapping a '10' for ten '1s' or a '100' for ten '10s' or vice versa (used in addition and subtraction when 'moving' 'ten' or a 'hundred' from its column into the next column and splitting it up). Heavily relied upon for addition and subtraction of larger numbers. Skills in this can be built up practically with objects, then Dienes rods/base 10, then place value counters before relying on a solely written method.

Expanded Multiplication – a method for multiplication where each stage is written down and then added up at the end in a column

Factor – a number that divides exactly into another number, without remainder

Grid method – a method for multiplying two numbers together involving partitioning and multiplying each piece separately.

Grouping – an approach to division where the dividend is split into groups of the size of the divisor and the number of groups created are then counted.

Half - a number, shape or quantity divided into 2 equal parts

Halve – divide a number by 2

Integer - a whole number (i.e. one with no decimal point)

Inverse – the opposite operation. For example, addition is the inverse of subtraction and multiplication is the inverse of division.

Known Multiplication Facts – times tables and other number facts that can be recalled quickly to support with larger or related calculations e.g. if you know 4x7 then you also know 40 x 70, 4 x 0.7 etc.

Long Division – formal written of division where the remainders are calculated in writing each time (extended version of short division)

Long Multiplication – formal written method of column multiplication

Multiple - a number which is an exact product of another number i.e. a number which is in the times table of another number

Number bonds – 2 numbers that add together to make a given total, e.g. 8 and 2 bond to 10 or 73 and 27 bond to 100
Number line – a line either with numbers or without (a blank numberline).
The number line emphasises the continuous nature of numbers and the existence of 'in-between' numbers that are not whole. It is based around the gaps between numbers.
Children use this tool to help them count on or count back for addition of subtraction. As they get older, children will count in 'jumps' on a number line e.g. to add 142 to a number they may 'jump' 100 and then 40 and then 2. The number line is sometimes used in multiplication and division but can be time consuming.
Number track – a sequence of numbers, each inside its own square. It is a simplified version of the number line that emphasises the whole numbers.
Numicon – practical maths equipment that teaches children the names and values of numbers 1-10 initially but them helps them with early addition, subtraction, multiplication and division. Numicon is useful for showing the real value of a number practically.
One-Step Calculation – a calculation involving only one operation e.g. addition. Usually the child must decide what that operation is.
Partition – split up a larger number into parts, such as the hundreds, tens and units e.g. 342 can be partitioned into 300 and 40 and 2
Place Value – the value of a digit created by its position in a number e.g. 3 represents thirty in 234 but three thousand in 3567
Recombine – for addition, once you have partitioned numbers into hundreds, tens and units then you have to add then hundreds together, then add the tens to that total, then add the units to that total
Remainder – a whole number left over after a division calculation
Repeated addition – repeatedly adding groups of the same size for multiplication
Scaling – an approach to multiplication whereby the number is 'scaled up' by a factor of the multiplier e.g. 4 x 3 means 4 scaled up by a factor of 3.
Sharing – an approach to division whereby the dividend is shared out into a given number of groups (like dealing cards)
Short Division - traditional method for division with a single digit divisor (this is a compact version of long division, sometimes called 'bus stop')
Significant digit – the digit in a number with the largest value e.g. in 34 the most significant digit is the 3, as it has a value of '30' and the '4' only has a value of '4'
Single digit – a number with only one digit. These are always less than 10.
Sum – the total of two or more numbers (it implies addition). Sum should not be used as a synonym for calculation.
Two-step calculation - a calculation where two different operations must be applied e.g. to find change in a shop you will usually have to add the individual prices and then subtract from the total amount. Usually the child has to decide what these two operations are and the order in which they should be applied.