Abingdon Primary School

Calculation Policy 2014

About our Calculation Policy

This policy shows how calculations are taught throughout Abingdon Primary School and has been produced to meet the statutory requirements of the National Curriculum 2014. It is designed to provide children with a clear progression though the calculation strategies.

Early learning of number and calculation in Foundation Stage follows the "Development Matters" EYFS document and this calculation policy is devised to follow on progressively from the content and methods established in the Early Years Foundation Stage.

As there is a greater emphasis on written methods and also the fact that they are introduced at an earlier age, it is vital that children are taught conceptual understanding through a wide range of resources in order to gain deeper understanding and fluency.

Year Group Expectations

The policy is organised into year groups and is based on expectation for that year group. However, it must be taken into account that some children will take longer to understand and that previous methods should first of all be consolidated, embedded and then once secure, should a child move on. Once a child is secure with a method, they should not quickly move onto the next stage but should be given plenty of opportunities to practise the methods, providing them with many examples of problem solving and reasoning in order to deepen their understanding and master methods.

Choosing a Calculation Method

In order to choose the most efficient method for the numbers in the calculation, children will be taught to and encouraged to go through the following process before carrying out a calculation:

Can I do this in my head using a mental calculation?

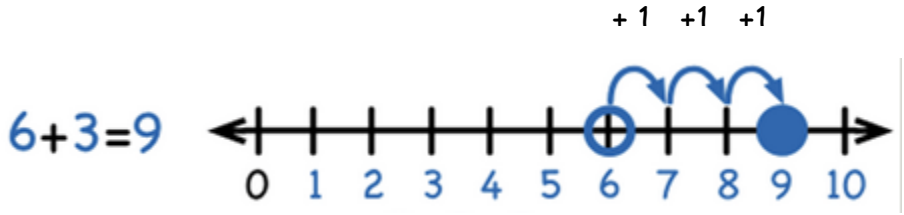
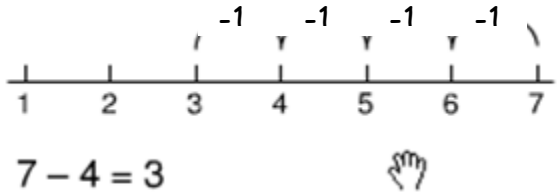
Will some jottings or a number line help?

Do I need to use a written method to do this?

To work out difficult calculations: estimate – calculate - check

Addition and Subtraction – Year 1

Key Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line , equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?

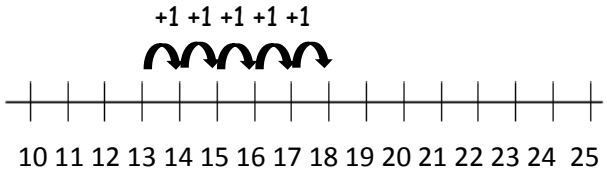
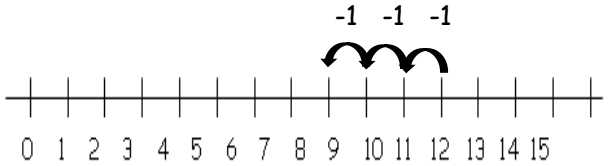
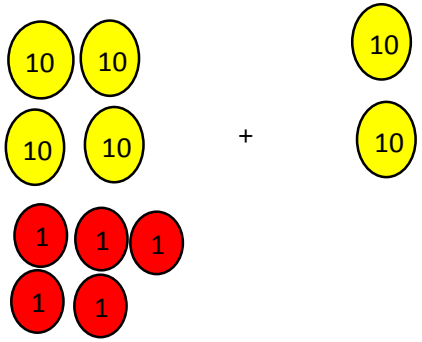
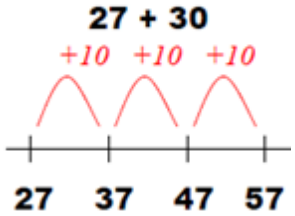
Recall/ mental	Representations	Problem Solving
Represent and use number bonds and related subtraction facts within 20 Add and subtract one-digit and two-digit numbers to 20, including zero **Written** Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs	All number bonds need to be taught, not just those to 10 or 20. Physical objects (counters, cubes, etc) can be used to put into groups, finding all different possibilities for each number, writing the number sentences alongside. Eg: Making 5  1 + 4 = 5, 2 + 3 = 5, 4 + 1 = 5, 3 + 2 = 5 The same can be done for subtraction facts. Use of fingers (number bonds to 10) and bead strings For addition and subtraction use manipulatives, such as: bead strings, cubes or counters, so that their experiences are concrete. Alongside these encourage them to write the appropriate number sentences. **Video Clip:** https://www.youtube.com/watch?v=OkW1Y11tGxw&list=UUVb98bWNgEmk02R7enUrmFA When children are ready to move away from manipulatives: Use numbered number lines to add, by counting on in ones. Encourage children to start with the **larger** number and count on. Eg  **Count back** in ones on a numbered number line to take away, with numbers up to 20:  It is really important that the children understand the equals sign as a sign of equivalence, that what is on one side of it has the same value as what is on the other. Record number sentences in different orders, such as 5 + 5 = 10 and 10 = 5 + 5	Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ - 9

Addition and Subtraction – Year 2

Key Vocabulary:

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, least, count back , how many left, how much less is_?

difference, count on, strategy, sum, tens, units, ones, partition, addition, column, tens boundary

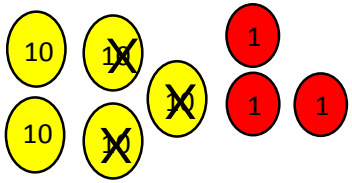
Objectives	Representations	Problem Solving
Recall/ Mental Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: * a two-digit number and ones * a two-digit number and tens * two two-digit numbers adding three one-digit numbers Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot	Continue to use concrete objects for number bonds until children are fluent. Video: https://www.youtube.com/watch?v=mEHKmMapWGY Children use numbered lines to support their own calculations using a numbered line to count on and back in ones. Eg 13 + 5 = 18  12 – 3 = 9  ***For adding and subtracting tens to and from a two-digit number***, use pictoral and concrete objects such as: number lines, Numicon, Diennes and place value counters to represent tens and ones (units). Children should gain an understanding that the ones digit does not change but the tens digit does. (Children should be secure in counting in tens) Eg: Using place value counters 45 + 20 = 65  Using number line  	Solve problems with addition and subtraction: * using concrete objects and pictorial representations, including those involving numbers, quantities and measures applying their increasing knowledge of mental and written methods *Solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change* (copied from Measurement)

Written

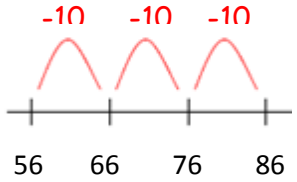
Read, write and interpret mathematical statements involving addition (+), subtraction (-), multiplication (x) and division (÷) and equals (=) signs

NB Objectives in black text are statutory requirements from the National Curriculum, objectives in green text are recommended and are from the Middlesbrough planning document.

53 – 30 = 33



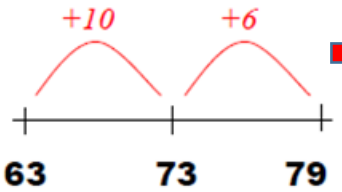
86 - 30



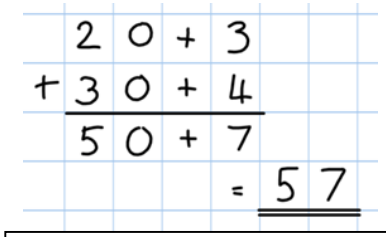
Add pairs of 2-digit numbers, moving to the partitioned column method ***when secure adding tens and units****:*

Using a number line

63 + 16

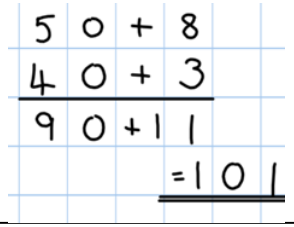


23 + 34



STEP 1: Only provide examples that do **NOT** cross the tens boundary until they are secure with the method itself.

58 + 43

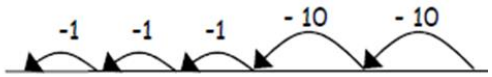


STEP 2: Once children can add a multiple of ten to a 2-digit number mentally (e.g. 80+11), they are ready for adding pairs

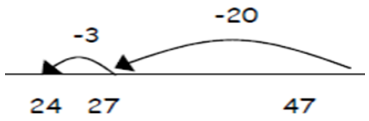
Subtracting pairs of two-digit numbers on a number line:

Eg:

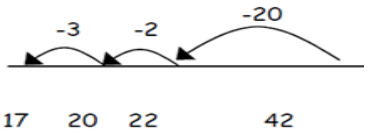
47 - 23 = 24 Partition the second number and subtract it in tens and units, as below:

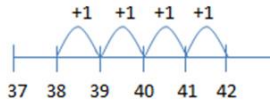


Move towards 24 25 26 27 37 47



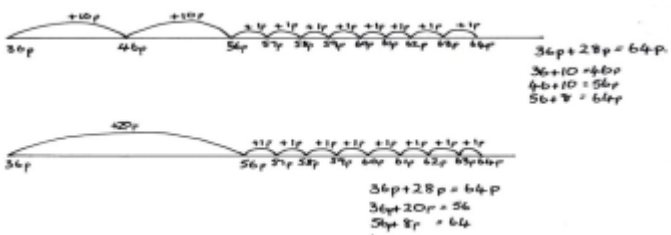
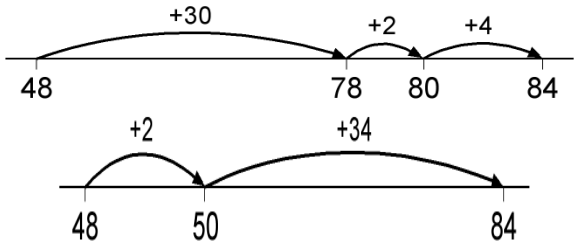
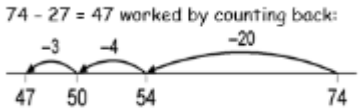
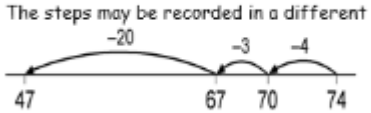
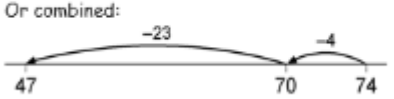
Teaching children to **bridge through ten** can help them to become more efficient, for example 42—25:



	Mental strategy - subtract numbers close together by **counting on:** Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to **count on** the difference. They need to be clear about the relationship between addition and subtraction. 42 – 38 = 4  It is really important that the children understand the equals sign as a sign of equivalence, that what is on one side of it has the same value as what is on the other. Record number sentences in different orders, such as 23 + 8 = 31 and 31 = 23 + 8	

Addition and Subtraction – Year 3

Key Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units(ones) partition, plus, addition, column, tens boundary, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, strategy, **exchange, decrease, hundreds, value, digit, hundreds boundary, increase, vertical, 'carry', expanded, compact**

Objectives	Representations	Problem Solving
Recall/ mental Recall and use addition and subtraction facts to 100 fluently, and derive and use related facts up to 1000 Add and subtract numbers mentally, including: * a three-digit number and ones * a three-digit number and tens * a three-digit number and hundreds Use their knowledge of the order of operations to carry out calculations involving the four operations	Use of the empty number line can be introduced to support mental calculations. Addition – Starting with the larger number to 'count-on': Eg I have 36p and my mum gives me 28p pocket money. How much money do I have altogether? N.B Only one number is partitioned here  Two examples of 48 + 36  Subtraction –    For addition, partitioning can be used as a mental method. Add the hundreds then the tens and finally the ones to form partial sums and then add these partial sums. It is not advised to use this method for subtraction as sometimes 'exchanging' is needed and this should be left for written methods.	Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

Written

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

NB Objectives in black text are statutory requirements from the National Curriculum, objectives in green text are recommended and are from the Middlesbrough planning document.

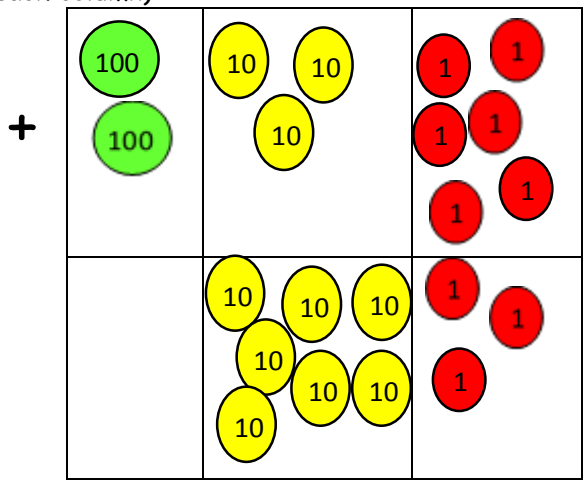
Example of addition:
124 + 276
100 + 200 = 300
20 + 70 = 90
4 + 6 = 10
Which is then recorded in a shorter form below
124 + 276 = 300 + 90 + 10 = 400

Written (addition)

Introduce the expanded column addition method, using place value counters or Diennes as concepts.

Eg: using paper folded into 3 columns or whiteboard with 3 columns, representing HTU

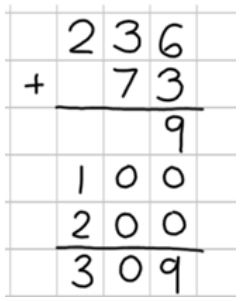
236 + 73
(Partition numbers and put correct number in each column)



Alongside this write:

Always add the units first, in preparation for the compact method!

2 3 6 (for how many counters in each column)
\+ 7 3
9 (counting in ones/units)
1 0 0 (counting in tens)
2 0 0 (counting in hundreds)
3 0 9



Most children will move away from a conceptual understanding to carrying out method without counters.

In order to carry out this method of addition:

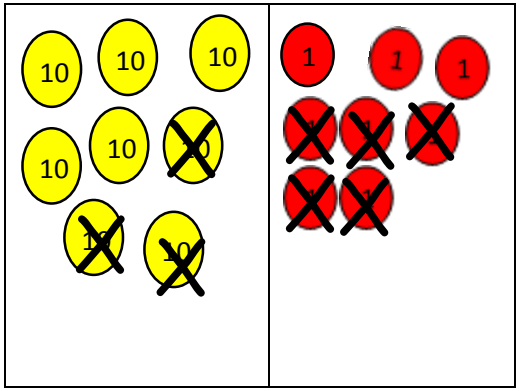
- Children need to recognise the value of the hundreds, tens and units without recording the partitioning.
- Pupils need to be able to add in columns.

Subtraction

For subtraction, using expanded column subtraction (with partitioning) method.
Introduce this method where no exchanging is required. Place Value counters can be used to introduce conceptual understanding.

Eg: 89 – 35

Step 1: Partiton 89 into 2 columns.



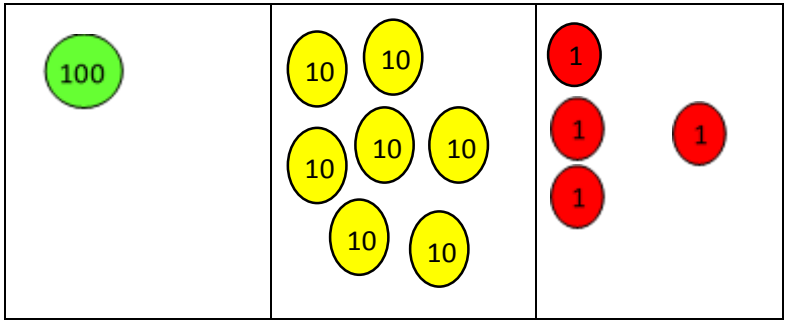
Step 2: Alongside this write

89 – 35 = **59**

80 + 9 (partitioning the number)
- 30 + 5
50 + 9 (Step 4)

Step 3: Subtract 3 tens and five ones

Once children are secure with this concept then they may move onto numbers that require exchanging.
Step 1: Partition 174 into 3 columns.

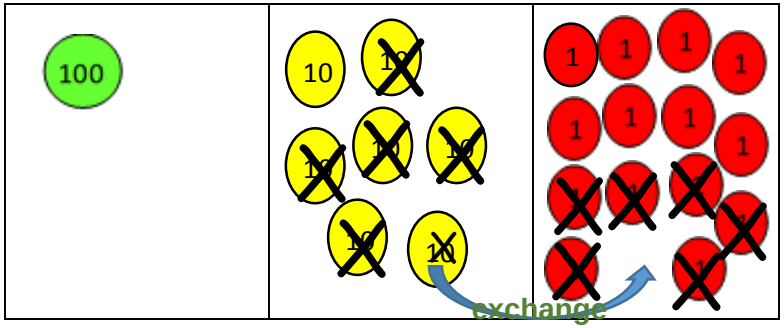


Step 2: Alongside this write

174 - 56

100 + 70 + 4 (partitioning the number)
- 50 + 6

Step 3: Because we cannot subtract six ones from the ones column we exchange a ten for ten ones.

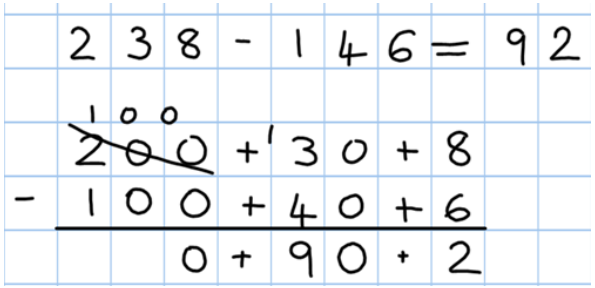


Step 4: Adjust step 2 algorithm

174 – 56 = 118
60 14
100 + ~~70~~ + ~~4~~
- 50 + 6
100 + 10 + 8

Step 5: Now subtract five tens and 6 ones.

Once pupils are secure with the understanding of "exchanging", they can use the partitioned column method to subtract any 2 and 3-digit numbers. Most children will move away from using place value counters and concrete objects into using the algorithm only.



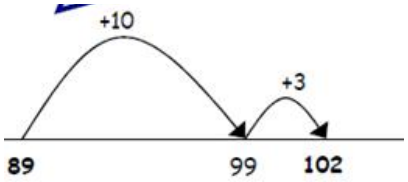
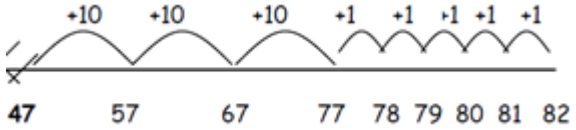
Videos:

Subtraction—teaching children to consider the most appropriate methods before calculating
http://www.youtube.com/watch?v=RCCLseBLBSo

Introducing partitioned column subtraction method, from practical to written
http://www.youtube.com/watch?v=dP8NlFLZzOg

Addition and Subtraction – Year 4

Key Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units (ones), partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, "carry", expanded, compact, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, least, count back , how many left, how much less is_? difference, count on, strategy, exchange, decrease, hundreds, value, digit, inverse thousands, hundreds, digits, inverse

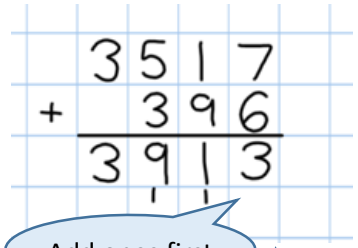
Objectives	Representations	Problem Solving
Recall/ Mental Recall and use addition and subtraction facts to 1000 fluently, and derive and use related facts up to 10000 Add and subtract numbers mentally, including: * a three-digit number and ones * a three-digit number and tens * a three-digit number and hundreds Use their knowledge of the order of operations to carry out calculations involving the four operations **Written** Add and subtract numbers with up to 4 digits using the formal written methods of	Continue to build on mental methods taught in Year 3. Children should be encouraged to decide whether a mental method or a written method would be most efficient for the calculation. **Counting on** as a mental strategy for subtraction: Continue to reinforce counting **on** as a strategy for **close-together numbers** (e.g. 121—118), and also for numbers that are "nearly" multiples of 10, 100, 1000 or £s, which make it easier to count on (e.g. 102-89, 131—79, or calculating change from £1 etc.). • Start at the smaller number and count on in **tens** first, then count on in units to find the rest of the difference:   	Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

columnar addition and subtraction where appropriate

NB Objectives in black text are statutory requirements from the National Curriculum, objectives in green text are recommended and are from the Middlesbrough planning document.

Written (Addition)

Move from expanded addition to the compact column method, adding units first, and "carrying" numbers underneath the calculation. Also include money and measures contexts.
e.g. 3517 + 396 = 3913

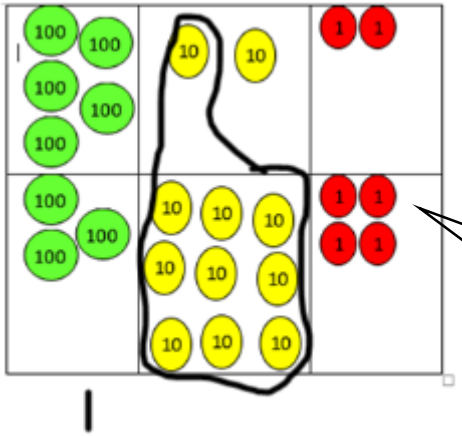


Introduce the **compact column addition** method by asking children to add the two given numbers together using the method that they are familiar with (expanded column addition—see Y3). Teacher models the compact method with carrying, asking children to discuss similarities and differences and establish how it is carried out.

"Carry" **numbers** underneath the bottom line.

Reinforce correct place value by reminding them the actual value is 5 hundreds add 3 hundreds, not 5 add 3, for example.

The use of place value counters and other concrete objects can be used alongside this in a similar way to Year 3.
Eg with 'carrying' numbers. 522 + 394 =



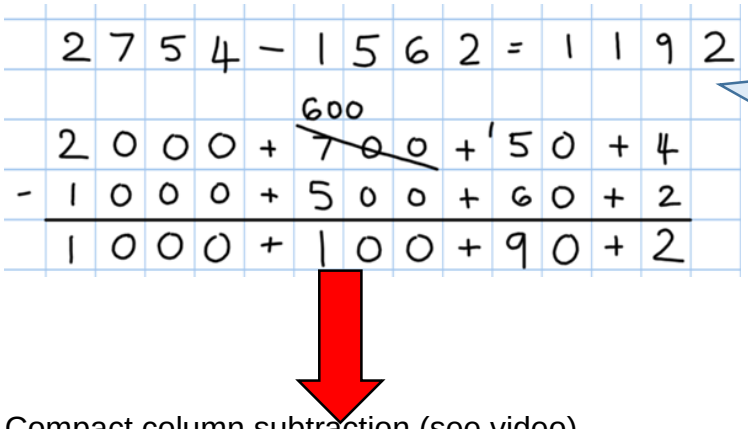
```
  522
+ 394
  916
  1
```

Alongside this, write the algorithm so that children can make connections.

Show the 1 group of 10, making 100 and being 'carried' to the hundreds.

(Subtraction)

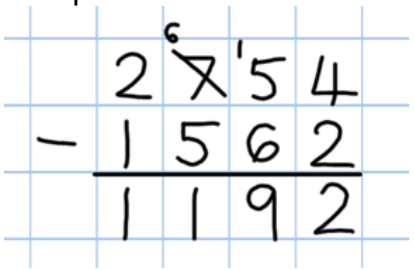
Expanded column subtraction with “exchanging” (decomposition):



As introduced in Y3, but moving towards more complex numbers and values. Use **place value counters** to reinforce “exchanging”.

Subtracting money: partition into £1 + 30 + 5 for example.

Compact column subtraction (see video)



To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask pupils to consider how it relates to the method they know, what is similar and what is different, to develop an understanding of it (shown on video).

- Give plenty of opportunities to apply this to money and measures.
- Always encourage children to consider the best method for the numbers involved—mental, counting on, counting back or written method (see video).

Videos:

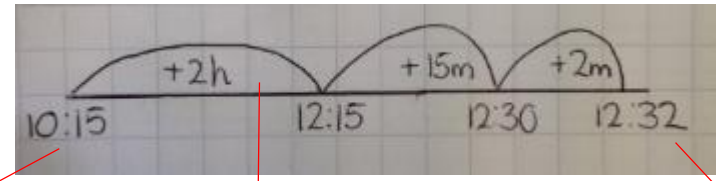
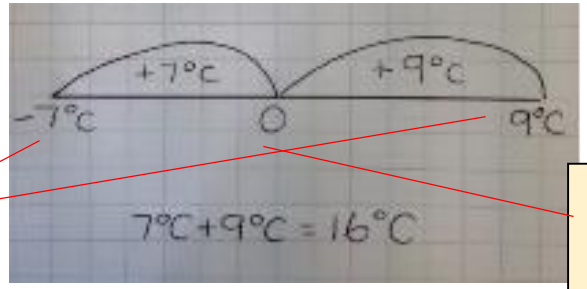
Subtraction—teaching children to consider the most appropriate methods before calculating
http://www.youtube.com/watch?v=RCCLseBLBSo

Introducing partitioned column subtraction method, from practical to written
http://www.youtube.com/watch?v=dP8NlFLZzOg

Moving to the compact column method of subtraction http://www.youtube.com/watch?v=3ihxp2mqnhs

Addition and Subtraction – Year 5

Key Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units (ones), partition, plus, addition, column, tens boundary, hundreds boundary, increase, "carry", expanded, compact, vertical, thousands, hundreds, digits, inverse, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, exchange, decrease, hundreds, value, digit, inverse, **tenths, hundredths, thousandths, decimal point, decimal, decimal places**

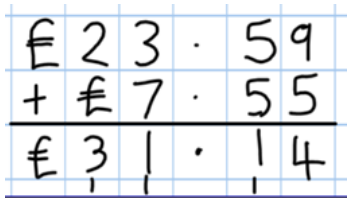
Objectives	Representations	Problem Solving
Recall/ Mental Recall and use addition and subtraction facts to 1000 fluently, and derive and use related facts up to 100000 Add and subtract numbers mentally with increasingly large numbers Use their knowledge of the order of operations to carry out calculations involving the four operations **Written** Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar	Children will become increasingly confident in deciding on most efficient method for the calculation depending on the numbers involved. Mental methods previously taught in Years 3 and 4 will be reinforced and the use of the empty number line should be encouraged when dealing with problems involving time and negative numbers. The use of the empty number line can also be useful for money word problems. Eg: A film starts at 10:15 and lasts for 2 hours and 17 minutes. What time does the film end?  Step 1: Write the start time at the left hand side of the empty number line Step 2: Add on 2 hours and 17 minutes by partitioning the minutes into 'easy' chunks. In this case 15m + 2m. Step 3: Look at the time at the end of the number line. This is the time that the film ends. The temperature in London is 9°C. The temperature in Moscow is -7°C. What is the difference in temperature?  Step 1: When finding 'difference' we put both values onto the empty numberline, always with the smaller value to the left. In this case -7°C and 9°C. Step 3: Add up the 'jumps'. In this case 7°C + 9°C = 16°C Step 2: Count on amounts that make it easy to bridge through numbers, in this case it is easy to bridge through zero by adding 7. From zero, 9 can be then added.	Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

addition and subtraction)

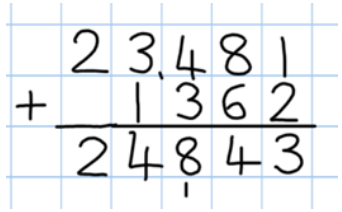
NB Objectives in black text are statutory requirements from the National Curriculum, objectives in green text are recommended and are from the Middlesbrough planning document.

Written (Addition)

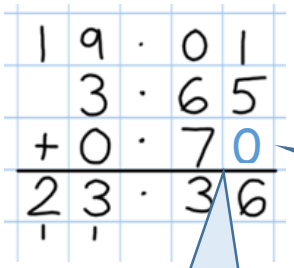
Add numbers with more than 4 digits
including money, measures and decimals with different numbers of decimal places.



The decimal point should be aligned in the same way as the other place value columns, and must remain in the same column in the answer row.



Numbers should exceed 4 digits.



Pupils should be able to add **more than two values**, carefully aligning place value columns.

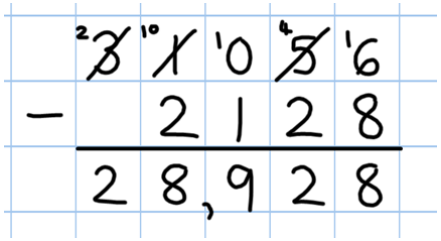
Empty decimal places can be filled with zero to show the place value in each column.

Say "6 tenths add 7 tenths" to reinforce place value.

Subtraction

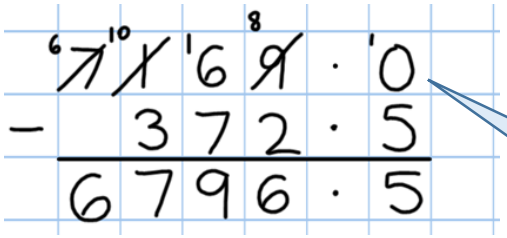
(including money, measures, decimals.)

Compact column subtraction
(with “exchanging”).



Subtracting with larger integers.

Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method. Place value counters can be used alongside this (see Year 4).



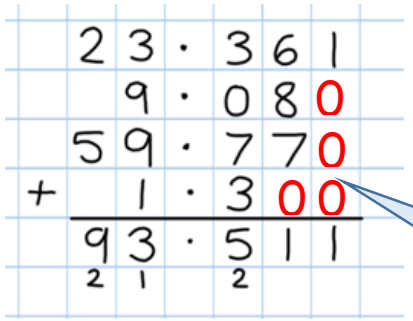
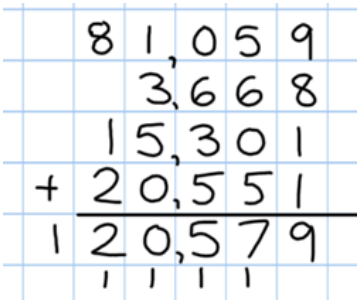
Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

Add a “zero” in any empty decimal places to aid understanding of what to subtract in that column.

Video:
Moving to the compact column method of subtraction http://www.youtube.com/watch?v=3ihxp2mqnhs

Addition and Subtraction – Year 6

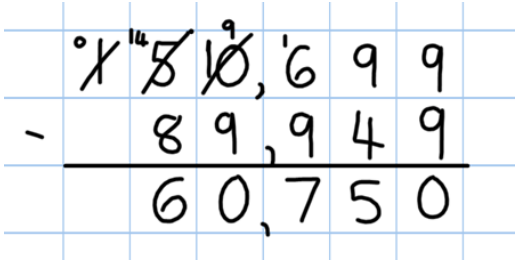
Key Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units (ones), partition, plus, addition, column, tens boundary, hundreds boundary, increase, „carry", expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, exchange, decrease, value

Objectives	Representations	Problem Solving
Recall/ Mental Recall and use addition and subtraction facts to 1000 fluently, and derive and use related facts up to 1000000 Perform mental calculations, including with mixed operations and large numbers Use their knowledge of the order of operations to carry out calculations involving the four operations **Written** Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)	Children should be given the opportunity to rehearse a range of mental calculations and they should do this with confidence and fluency. The quickly decide on the most efficient method for the numbers involved. Reinforce the use of the empty number line for calculations involving time, negative numbers and some money word problems. Children should be encouraged to use a checking strategy (Eg: calculating using a different method or using the inverse). **Written (Addition)** **Add several numbers of increasing complexity**  Adding several numbers with different numbers of decimal places (including money and measures): • Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row. Empty decimal places should be filled with zero to show the place value in each column.  Adding several numbers with more than 4 digits.	solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why Solve problems involving addition, subtraction, multiplication and division

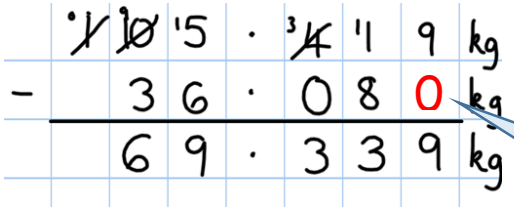
NB Objectives in black text are statutory requirements from the National Curriculum, objectives in green text are recommended and are from the Middlesbrough planning document.

(Subtraction)

Subtracting with increasingly large and more complex numbers and decimal values.



Using the compact column method to subtract more complex integers



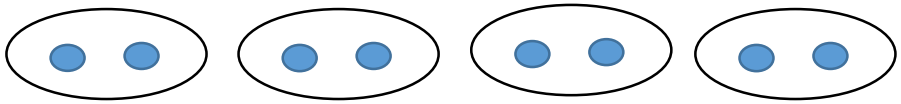
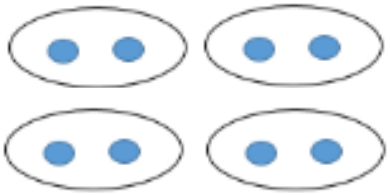
Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.

Empty decimal places should be filled with zero to show the place value in each column.

See videos in Years 4 and 5 to show understanding of compact method.

Multiplication and Division – Year 1

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, share, share equally, one each, two each…, group, groups of, lots of

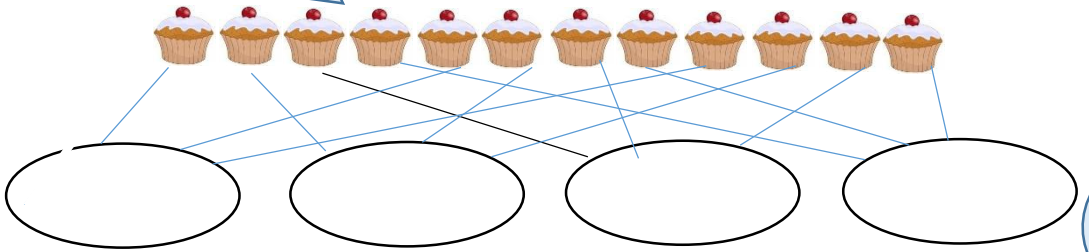
Objectives	Representations	Problem Solving
Recall/ Mental Count in multiples of twos, fives and tens (From Number and Place Value)	Use concrete objects, pictorial representations and arrays. **Multiplication word problems** Eg: How many fingers do 2 boys have altogether?  5 10 15 20 There are 2 sweets in each bag. How many sweets are there altogether?  Concrete objects could be put into arrays to enable children to make further connections. Eg  2 x 4 = 8	Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Division word problems

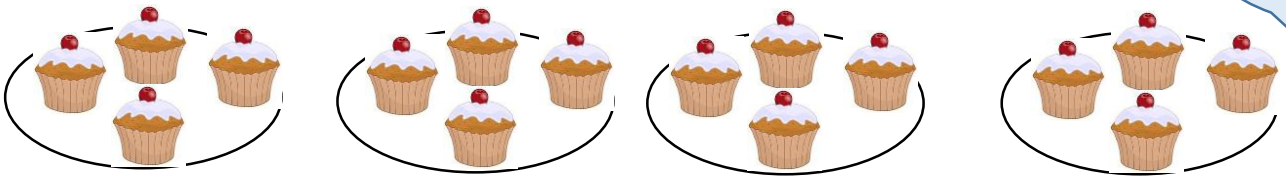
Begin by using sharing for division. Remainders can be introduced once children are secure.
Division word problems.
Eg: Sam made 12 cakes for his 4 friends. How many cakes did each friend get?

Step 1 – Count out 12 cakes

Step 2 – share the cakes between 4 (plates could be used)

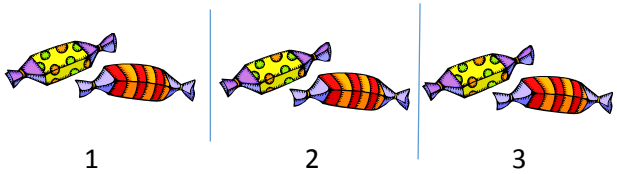


Step 3 – Count the number of cakes on each plate



Once children are secure with sharing then grouping can be introduced. Ensure correct vocabulary is used so misconceptions are not introduced.

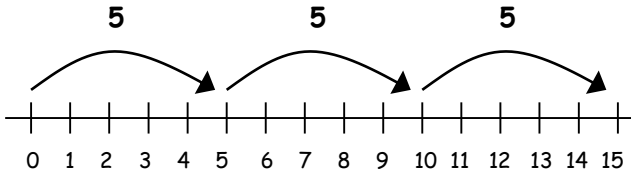
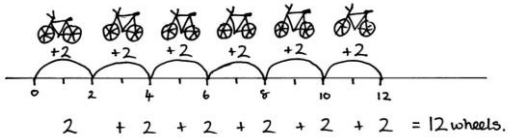
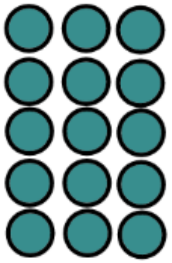
Eg: How many groups of 2 can be made from these sweets?



Children put the sweets into groups of 2 and then count how many groups there are.

Multiplication and Division – Year 2

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, **share, share equally, one each, two each…, group, equal groups of,** multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times… **divide, divided by, divided into, division, grouping, number line, left, left over**

Objectives	Representations	Problem Solving
Recall/ Mental count in steps of 2, 3, and 5 from 0, and in tens from any number, forward or backward (from Number and Place Value) recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot **Written** Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×),	Children should begin to recall multiplication facts for 2, 5 and 10 times tables through practice in counting and understanding of the operation. **Multiply using arrays and repeated addition (using at least 2s, 5s and 10s)** Eg: 3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3 Repeated addition can be shown easily on a number line. **5 x 3 = 5 + 5 + 5**  Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs. If I have 6 bicycles how many wheels would there be?  6 x 2 = 12 **Arrays** **Eg:**  **5 x 3** = 3 + 3 + 3 + 3 = **15** **3 x 5** = 5 + 5 + 5 = **15**	solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

division (÷) and equals (=) signs

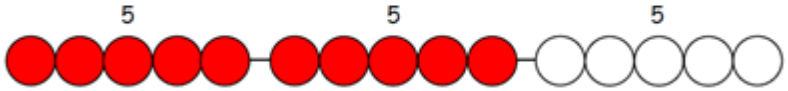
To represent division as repeated subtraction

To record division calculations with remainders using a number line
(TU ÷ U)

Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as **3 x __ = 6.**

Use practical apparatus:

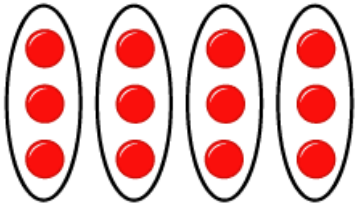
5 x 3 = 5 + 5 + 5



Division

Group and share, using the ÷ and = sign. Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Using Arrays



12 ÷ 3 = 4

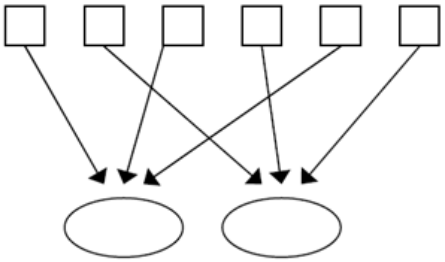
This represents 12 ÷ 3, posed as: how many groups of 3 are in 12?

Pupils should also show that the same array can represent 12 ÷ 4 = 3 if grouped horizontally

Know and understand sharing and grouping:

6 sweets shared between 2 people, how many do they each get?

Children should be taught to recognise whether problems require sharing or grouping.



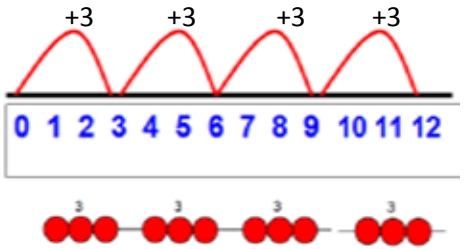
sharing

There are 6 sweets, how many people can have 2 sweets each?

grouping

Grouping using a number line:

Group from zero in equal jumps of the divisor to find out "how many groups of _ in _ ?". Pupils could and using a bead string or practical apparatus to work out problems like „A CD costs £3. How many CDs can I buy with £12?" **This is an important method to develop understanding of division as grouping.**



Pose **12 ÷ 3** as "**How many groups of 3 are in 12?**"

12 ÷ 3 = 4

Videos:

Multiple Representations of Multiplication
https://www.youtube.com/watch?v=YPWmOVt8vgw&list=UUVb98bWNgEmk02R7enUrmFA

The Commutative Law for Multiplication
https://www.youtube.com/watch?v=VGkjjVfnGYI&list=UUVb98bWNgEmk02R7enUrmFA

Sharing and Grouping (whole class)
http://vimeo.com/83485518

Sharing and Grouping (pairs)
http://vimeo.com/83485658

Multiplication and Division – Year 3

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times..., share, share equally, one each, two each..., group, equal groups of, divide, divided by, divided into, division, grouping, number line, left, left over, **partition, grid method, multiple, product, tens, units, value, inverse, short division, 'carry', remainder, multiple**

<table>
<tr><th>Objectives</th><th>Representations</th><th>Problem Solving</th></tr>
<tr>
<td>Recall/ Mental
count from 0 in multiples of 4, 8, 50 and 100
(copied from Number and Place Value)

recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables

write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods (appears also in Written Methods)</td>
<td>Introduce the grid method for multiplication of TU x U.

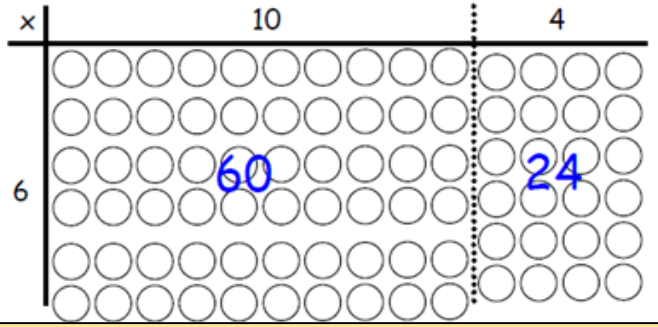
Eg 14 x 6

<table>
<tr><td>X</td><td>10</td><td>4</td></tr>
<tr><td>6</td><td>60</td><td>24</td></tr>
</table>

60 + 24 = 84

To begin with, children should be encouraged to link a multiplication calculation to an array. This knowledge will support with the development of the grid method.

Introduce the grid method with children physically making an array to represent the calculation (e.g. make 6 lots of 14 with 10s and 1s place value counters), then translate this to grid method format (see video clip).

Children will need to be secure with partitioning to be able to carry this out successfully. They will need to be able to multiply multiples of 10 by a single digit (eg 30 x 3).

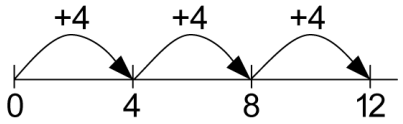
For multiplication facts not known they should use repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays:</td>
<td>solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects</td>
</tr>
</table>

show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot

Written

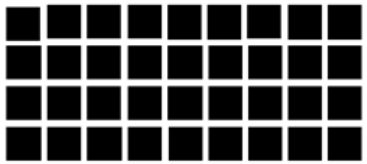
write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

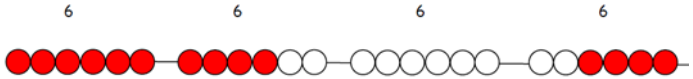
To record division calculations with remainders using a number line
(TU ÷ U)
(HTU ÷ U)



Videos:

Multiplication – Lower Key Stage 2





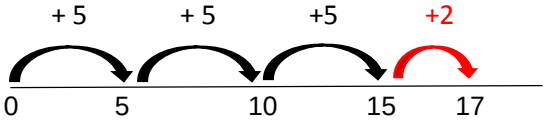
https://www.youtube.com/watch?v=qyTRtoqYi7Q&list=PLQqF8sn28L9yj34NpXK7Yffze7ZoXTiix

Division:

Grouping on a number line

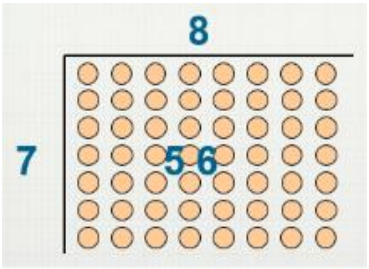
Children continue to work out unknown division facts by using repeated addition on a number line.

Eg
17 ÷ 5 = 3 r 2



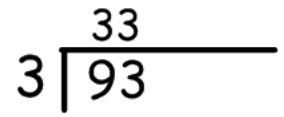
They are also now taught the concept of **remainders**, as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s, ready for "carrying" remainders across within the short division method.

Using Arrays
Eg 56 ÷ 7 = 8



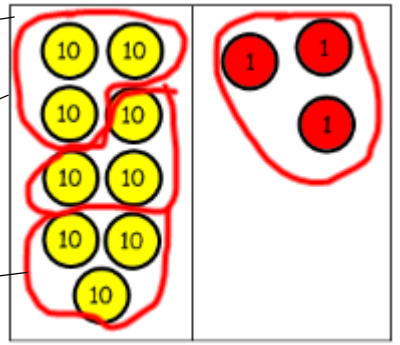
Short Division
Eg 93 ÷ 3 =
Once children are secure with grouping and can demonstrate on number lines, they are then able to move onto short division. This should be introduced in a visual way, using place value counters and where there is no remainder.

Step 1 – Draw the bus stop with the number that is 'divided by' (divisor) on the outside and the number that is being divided (dividend) on the inside.



Step 2 – Under this draw the number of columns needed for the calculation (in this case 2). Use place value counters to partition 93 into 9 tens and 3 units.

Step 3 – Group the place value counters into the number that is being divided by (in this case 3).

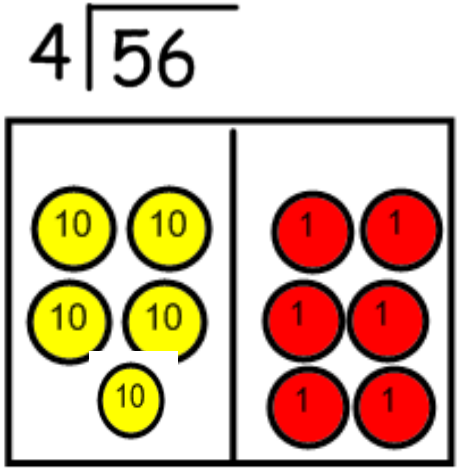


Step 4 – Write the number of groups in the written algorithm.

Once children are secure with short division and have a full understanding of remainders then they can move onto short division with remainders within the calculation but not in the final answer,

Eg 56 ÷ 4

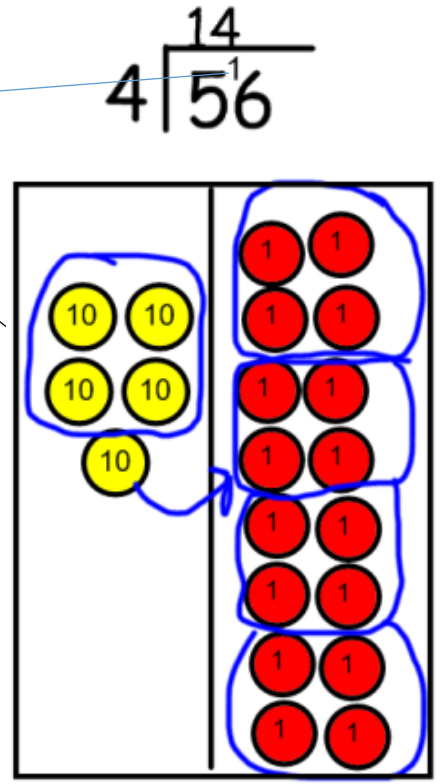
Carry out steps 1 and 2 as in previous example.



In step 3, children will need to ‘exchange’ a ten for ten ones.

This should be shown in the written algorithm as the remainder being ‘carried’ into the next digit.

If needed, children should use the number line to work out individual division facts that occur which they are not yet able to recall mentally.



Videos: (Y4 examples but can be adapted to suit Year 3)
Representing division with place value counters **http://vimeo.com/83485661**
Using place value counters and recording division http://vimeo.com/83485662

Multiplication and Division – Year 4

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times as big as, once, twice, three times… partition, grid method, total, multiple, product, sets of, share, share equally, one each, two each…, group, equal groups of, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, "carry", remainder, multiple, inverse, divisible by, factor

Objectives	Representations	Problem Solving
Recall/ Mental *Count in multiples of 6, 7, 9, 25 and 1 000* (copied from Number and Place Value) Recall multiplication and division facts for multiplication tables up to 12 × 12 Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers Recognise and use factor pairs and commutativity in mental calculations (appears also in Properties of Numbers)	**Multiplication** Continue to develop the grid method for multiplication of TU X U and HTU x U (Use Place Value Counters for arrays if needed – see Y3) Eg. 136 x 5 = 680 Encourage children to use a column to add correctly X | 100 | 30 | 6 5 | 500 | 150 | 30 500 150 + 30 680 When children are secure with the grid method, introduce short multiplication. Step 1 – expanded short multiplication. This enables the child to represent the method of recording in a column format, but showing the working. Establish links between this and the grid method. Eg 38 x 7 = 266 38 X 7 56 210 266 Step 2 – short multiplication. Only when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit using expanded short multiplication, **and** are already confident in "carrying" for written addition, should they be moved onto this method. Eg 327 x 4 = 1308 3 2 7 x 4 1 3 0 8 1 2	solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Written

Multiply two-digit and three-digit numbers by a one-digit number using formal written layout

To record division calculations using formal written method with remainders (HTO ÷ O)

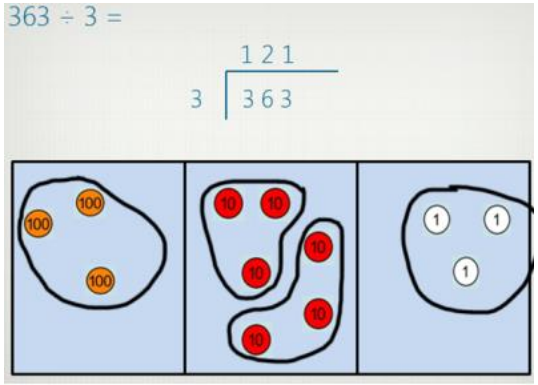
Division

Continue to develop short division. (see Year 3 for individual steps)

Pupils must be secure with the process of short division for dividing 2-digit numbers by a single digit (**those that do not result in a final remainder** —see steps in Y3), but must understand how to calculate remainders, using this to "carry" remainders within the calculation process.

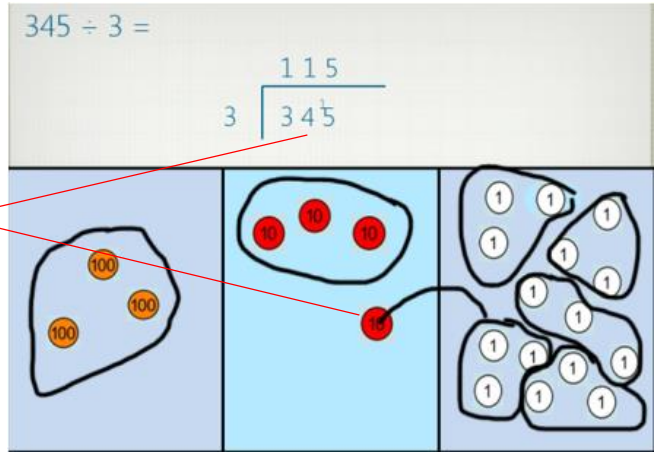
Pupils move onto dividing numbers with up to **3-digits** by a single digit, however problems and calculations provided should **not result in a final answer with remainder** at this stage**.**

Example without remainders within calculation:

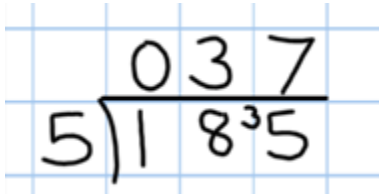


Example with remainders within calculation:

Remember to 'exchange' a ten for ten ones and show this in written algorithm as 'carrying' to the next digit as a remainder.



When the answer for the **first column** is zero (1 ÷ 5, as in example), children could initially write a zero above to acknowledge its place, and must always "carry" the number (1) over to the next digit as a remainder.



Videos:

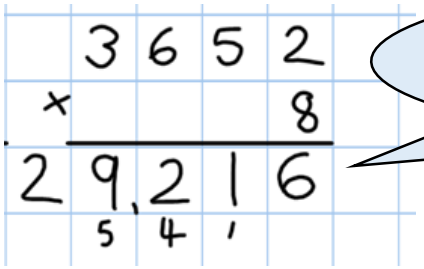
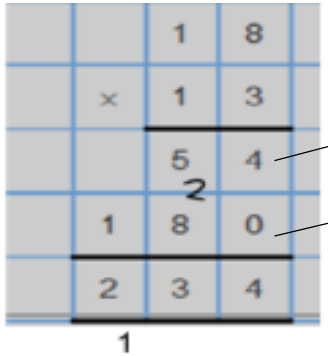
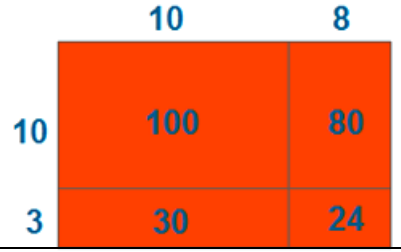
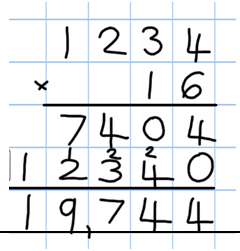
Multiplication – Lower Key Stage 2 (links to place value counters as shown in Y3) http://vimeo.com/70319240

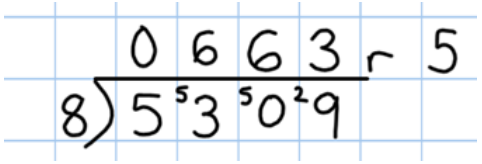
Representing division with place value counters **http://vimeo.com/83485661**

Using place value counters and recording division http://vimeo.com/83485662

Multiplication and Division – Year 5

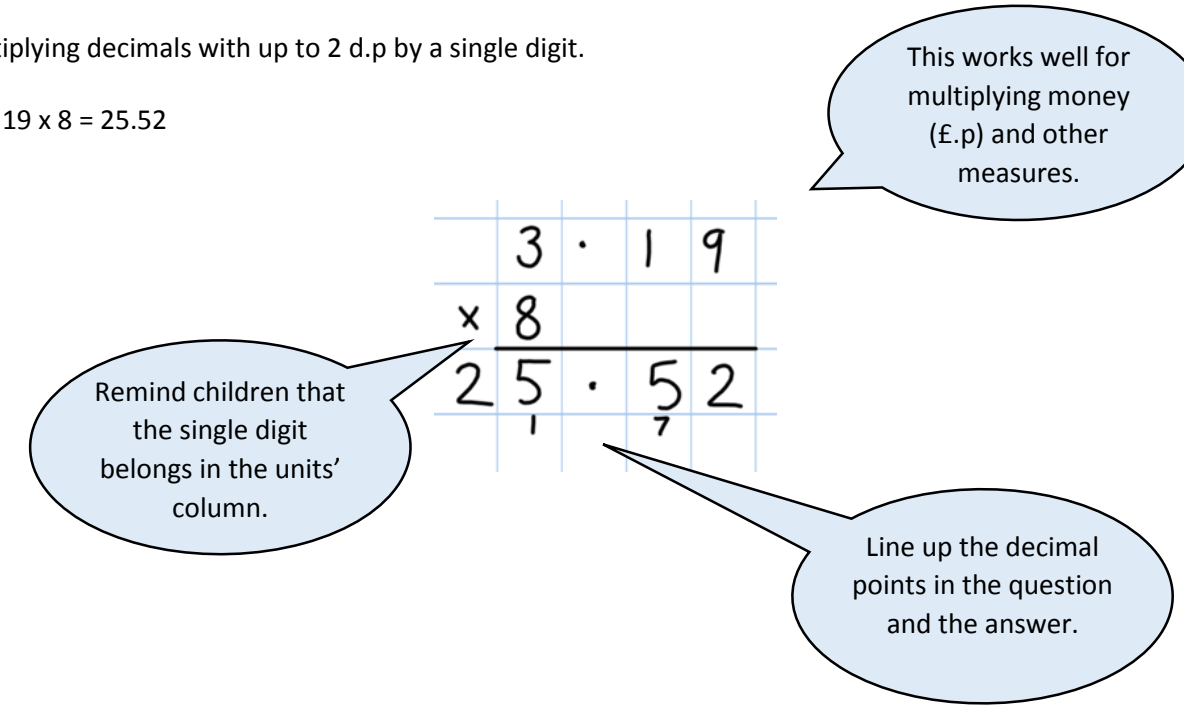
Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times…, partition, grid method, total, multiple, product, inverse, share, share equally, one each, two each…, group, equal groups of, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, "carry", remainder, multiple, divisible by, factor, inverse, square, factor, integer, decimal, short/long multiplication, 'carry', quotient, prime number, prime factors, composite number (non-prime)

Objectives	Representations	Problem Solving
Recall/ Mental Use multiplication and division facts (12x12) to be able to multiply and divide multiples of 10. Multiply and divide numbers mentally drawing upon known facts Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 **Written** Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and	**Multiplication** - Continue to develop short multiplication (see Year 4) for multiplication by a one-digit number. Eg 3652 x 8 = 29 216  To be successful when calculating in this way, it is good practice for children to estimate first. An example in this case is: 3652 is between 3000 and 4000, therefore 8 x 3000 = 24 000and 8 x 4000 = 32 000, The answer must be between 24 000 and 32 000. For multiplication by a 2-digit number, long multiplication can be introduced. Remember to estimate. 18 x 13 lies between 18 x 10 and 18 x 20. 10 x 18 is 180, 20 x 18 is 360. The answer must be between 180 and 360.  First row. Say 3 x 8 = 24. Carry the 2 for twenty, then 3 x 1 = 3, add the carried 2 which is 5. Second row. Say this is 10 x the top number so we always put a zero down first in the units column. Then 1 x 8 = 8. 1 x 1 = 1. (The grid could be used to introduce long multiplication, as the relationship can be seen in the answers in each row.)  Moving onto more complex numbers.  	solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates

interpret remainders appropriately for the context	**Videos:** Multiplication – Upper Key Stage 2 http://vimeo.com/70318365 Rapid Recall of Multiplication Facts https://www.youtube.com/watch?v=BcIjRLZzMaw&list=PLQqF8sn28L9wjDm8uJEJcRCDDoY6raPE_&index=2 **Division** **Short division with remainders:** Now that pupils are introduced to examples that give rise to remainder answers, division should have a real life problem solving context, where **pupils consider the meaning of the remainder and how to express it,** ie. as a fraction, a decimal, or as a rounded number or value , depending upon the context of the problem.  The answer to 5309 ÷ 8 could be expressed as 663 and five eighths, 663 r 5, as a decimal, or rounded as appropriate to the problem See Y6 for how to carry through short division to give a decimal answer for those who are confident.	

Multiplication and Division – Year 6

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times… partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, "carry", equal groups of, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, "carry", remainder, multiple, divisible by, factor, inverse, tenths, hundredths, decimal, common factor, common multiple

Objectives	Representations	Problem Solving
Recall/ Mental Use multiplication and division facts (12x12) to derive decimal multiplication and division facts. Perform mental calculations, including with mixed operations and large numbers *Associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. $^3/_8$)* (copied from Fractions) Multiply one-digit numbers with up to two decimal places by whole numbers (copied from Fractions – also includes written methods)	**Multiplication** Short and long multiplication as in Year 5. Multiplying decimals with up to 2 d.p by a single digit. Eg 3.19 x 8 = 25.52  **Videos**: Rapid Recall of Multiplication Facts https://www.youtube.com/watch?v=BcIjRLZzMaw&list=PLQqF8sn28L9wjDm8uJEJcRCDDoY6raPE_&index=2	solve problems involving addition, subtraction, multiplication and division *solve problems involving similar shapes where the scale factor is known or can be found* (copied from Ratio and Proportion)

Written

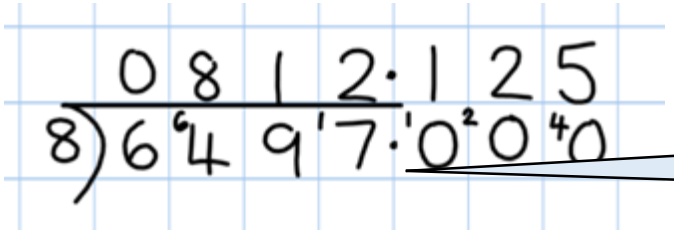
Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

Divide numbers up to 4-digits by a two-digit whole number using the formal written method of short division where appropriate for the context divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

Use written division methods in cases where the answer has up to two decimal places (copied from Fractions (including decimals))

Division

Short Division:



Add a decimal point after the units if there is still a remainder.

Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.
Calculating a decimal remainder: In this example, rather than expressing the remainder as r 1, a decimal point is added after the units because there is still a remainder, and the one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.

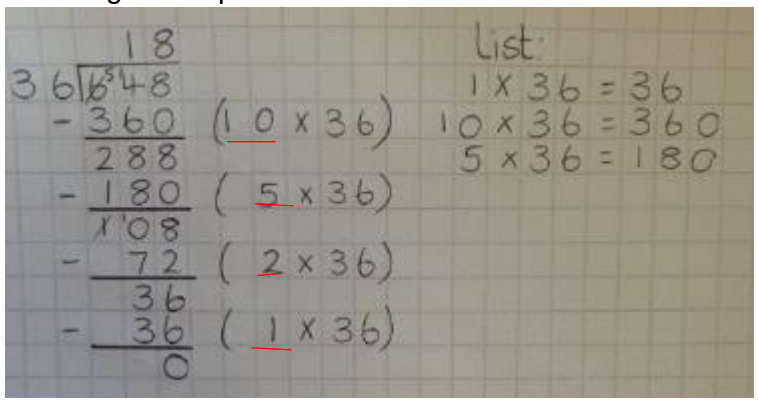
Long Division

Long Division by chunking for dividing by 2 digits:

Introduce chunking.
Eg: 648 ÷ 36 Step 1: estimate: 10 - 20

36 x 10 = 360
36 x 20 = 720
648 is between 360 and 720 so the estimate is between 10 and 20.

(Estimating will help children to reduce the number of subtractions being made.)



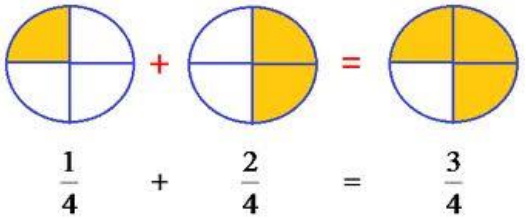
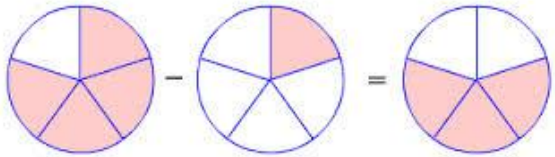
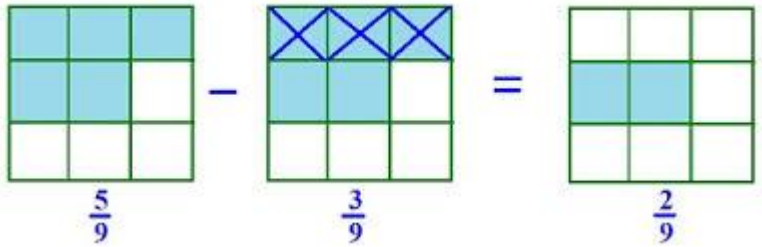
Step 2: Create a 'useful' list to help with subtractions. In this example we know the answer lies between 10 and 20 so there is no need to go up to 20 x 36. If we know 10 x 36 then we know 5 x 36 is half of this amount.

Step 3: Begin taking chunks of 36 away. Use useful list to help. Write in brackets how many 'lots' are being subtracted (always put the number of lots first then the number being multiplied).

Step 4: Count up how many 'lots' or 'chunks' of 36 have been subtracted. Write the answer above the division box.
Where remainders occur, pupils should express them as fractions, decimals or use rounding, depending upon the problem.

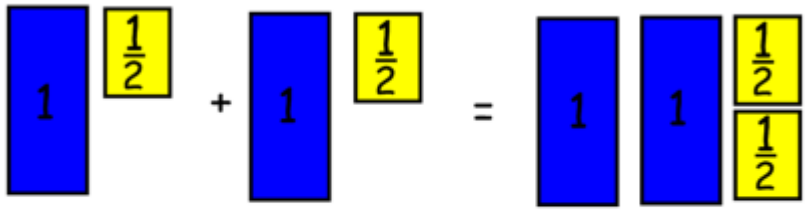
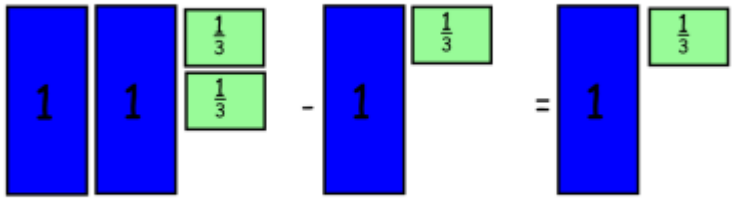
Calculation of Fractions – Year 3

Key Vocabulary: fraction, one whole, half, quarter, three-quarters, add, subtract, equal parts of a whole, numerator, denominator

Objectives	Representations	Problem Solving
Add and subtract fractions with the same denominator within one whole (e.g. $^5/_7 + {}^1/_7 = {}^6/_7$)	Use a variety of resources such as number rods, paper strips, equivalence circles, cards to model what happens when you add and subtract fractions with the same denominator. Children should be using concrete objects alongside the written fractions in order to gain conceptual understanding. Eg addition   Eg subtraction 4/5 – 1/5 = 3/5   **Videos (Key Stage 1 but objectives apply to Year 3):** Representing Fractions http://vimeo.com/83486102 Adding and subtracting fractions (Goes beyond one whole but examples can be adapted) http://vimeo.com/83486226 Reasoning about addition and subtraction of fractions http://vimeo.com/83486224	Solve problems involving fractions and decimals.

Calculation of Fractions – Year 4

Key Vocabulary: fraction, whole-one/s, half, quarter, three-quarters, numerator, denominator, add, subtract, equal parts of a whole, mixed number, equivalence, equivalent

Objectives	Representations	Problem Solving
Add and subtract fractions with the same denominator	Continue to develop addition and subtraction of fractions as shown in Year 3 but beyond the whole one, using lots of practical resources such as number rods, equivalence circles, cards, etc to ensure conceptual understanding. Eg Addition  $1\frac{1}{2} + 1\frac{1}{2} = 2\frac{2}{2}$ (most children should recognise equivalence that 2 halves are the same as one whole, therefore the answer is 3) $= 3$ Eg subtraction  $2\frac{2}{3} - 1\frac{1}{3} = 1$ **Videos (Key Stage 1 but objectives apply to Year 3):** Representing Fractions http://vimeo.com/83486102 Adding and subtracting fractions (Goes beyond one whole but examples can be adapted) http://vimeo.com/83486226 Reasoning about addition and subtraction of fractions http://vimeo.com/83486224 Key Stage 2 (Year 4) Developing Fluency – Counting in fractional steps http://vimeo.com/83486434 Preparing to add fractions with different denominators (has some ideas for consolidating adding with same denominator) http://vimeo.com/83486557	solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number solve simple measure and money problems involving fractions and decimals to two decimal places.

Calculation of Fractions (addition and subtraction) Year 5

Key Vocabulary: fraction, whole-one/s, half, quarter, three-quarters, numerator, denominator, add, subtract, equal parts of a whole, mixed number, equivalence, equivalent, array, model, improper fraction, multiples, common denominator

Objectives

Add and subtract fractions with the same denominator and multiples of the same number

Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number (e.g. $^{2}/_{5} + {}^{4}/_{5} = {}^{6}/_{5} = 1\,{}^{1}/_{5}$)

Representations

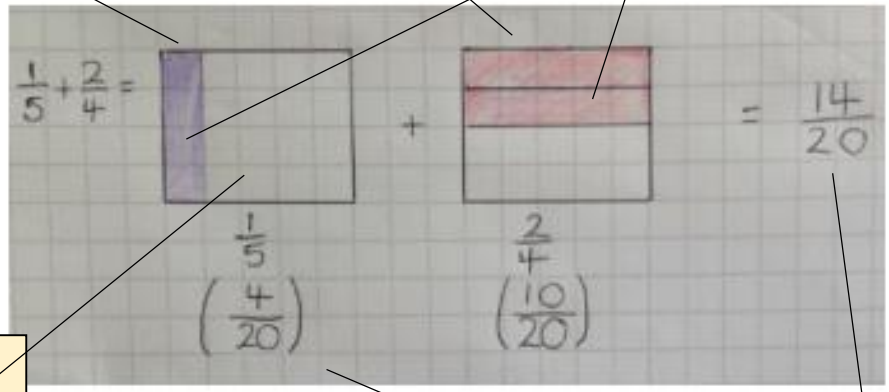
To gain a real secure understanding of adding and subtracting fractions with different denominators, children should be introduced to the array model so that conceptual understanding is embedded first. This array model can also be used for the multiplication and division of fractions with different denominators. As a school, this will be the only model that children should be shown so that possible misconceptions and confusion of using too many models is avoided.

The video listed shows how to introduce the model and the example below shows the steps children need.

Step 1: create a grid that can be split into fifths and quarters (count along 5 and down 4). Ensure correct vocabulary is used: numerator, denominator, equal parts, etc)

Step 2: draw an identical grid next to it with the + sign between.

Step 3: colour 1/5 of the first grid and 2/4 of the second grid.



Step 4: Find the denominator by counting how many equal parts the grid has been divided into, find the numerator by counting how many equal parts are shaded.

Step 5: Now the denominators are equal (common denominator), the addition of the fraction can be carried out.

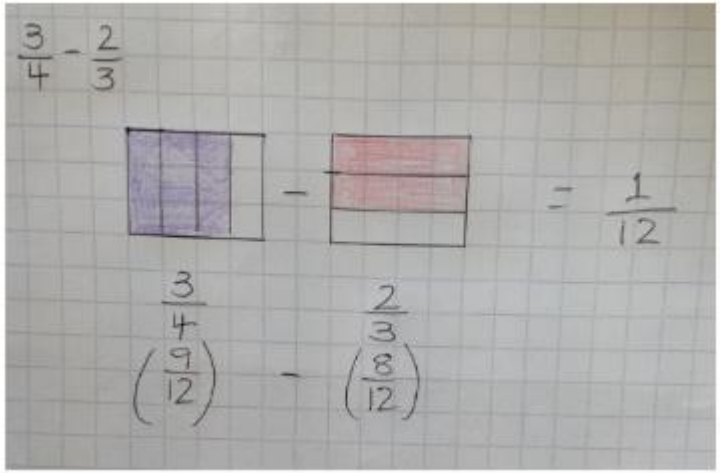
Problem Solving

Solve problems which require knowing percentage and decimal equivalents of $^{1}/_{2}$, $^{1}/_{4}$, $^{1}/_{5}$, $^{2}/_{5}$, $^{4}/_{5}$ and those with a denominator of a multiple of 10 or 25.

Subtraction:

Follow steps 1 – 4 from addition. Step 5 is a subtraction calculation.

Example:

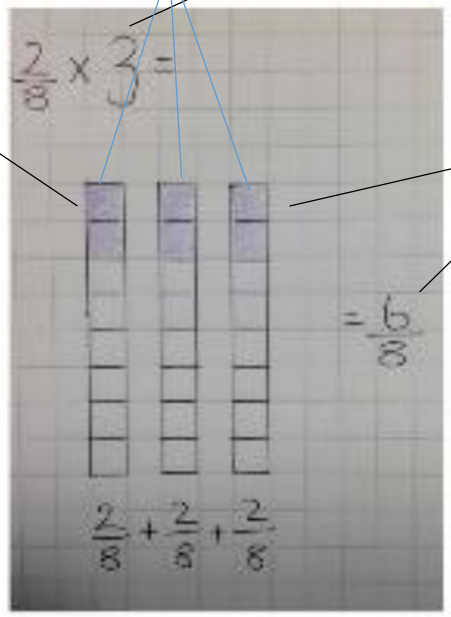


Videos:

Preparing to add fractions with different denominators (has some ideas for consolidating adding with same denominator)
http://vimeo.com/83486557
Adding fractions with different denominators (shows how to introduce the array model)
http://vimeo.com/83486433
Children exploring addition of fractions with different denominators
http://vimeo.com/83486338
Identifying Misconceptions
http://vimeo.com/83486435

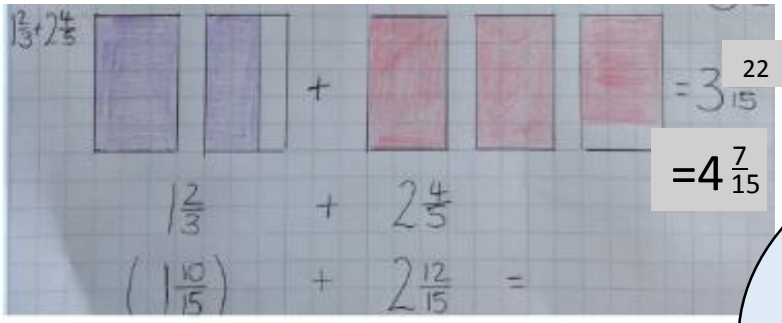
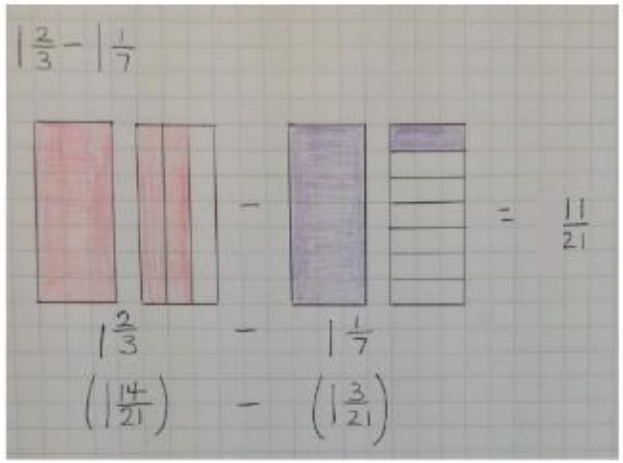
Calculation of Fractions (multiplication and division) – Year 5

Key Vocabulary: fraction, whole-one/s, half, quarter, three-quarters, numerator, denominator, equal parts of a whole, mixed number, equivalence, equivalent, array, model, improper fraction, common denominator, multiples, multiply, repeated addition

Objectives	Representations	Problem Solving
Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams	A similar model can be used for repeated addition for multiplication of fractions and mixed numbers by whole numbers. Eg Step 1: draw a grid that shows eighths of a whole. Shade in 2 parts of the whole (2/8). Step 2: look at the whole number that the fraction is being multiplied by (in this case 3). Draw the same grid so that it is repeated 3 times. Step 3: count how many shaded parts of the wholes there are and add these together.  	solve problems which require knowing percentage and decimal equivalents of $^{1}/_{2}$, $^{1}/_{4}$, $^{1}/_{5}$, $^{2}/_{5}$, $^{4}/_{5}$ and those with a denominator of a multiple of 10 or 25.

Calculation of Fractions (addition and subtraction) Year 6

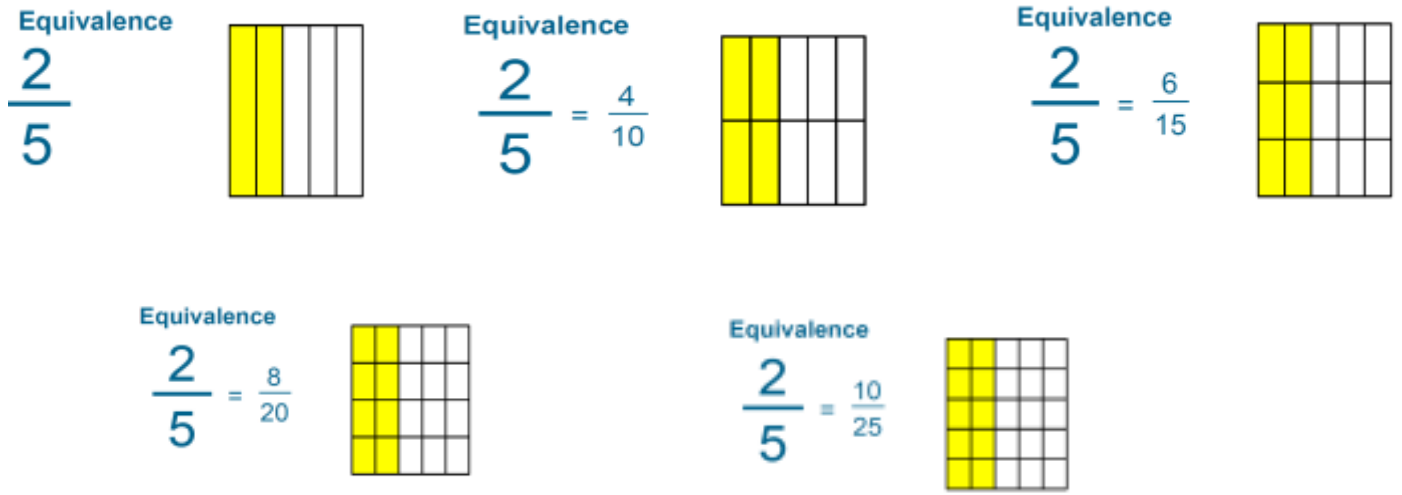
Key Vocabulary: fraction, whole-one/s, half, quarter, three-quarters, numerator, denominator, add, subtract, equal parts of a whole, mixed number, equivalence, equivalent, array, model, improper fraction, multiples, common denominator

Objectives	Representations	Problem Solving
Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions	Continue to use the array model taught in Year 5 to add fractions with different denominators. Introduce addition of mixed number fractions using the same array model. Examples: addition (see Y5 steps)  Subtraction  	solve problems which require knowing percentage and decimal equivalents of 1/2, 1/4, 1/5, 2/5, 4/5 and those with a denominator of a multiple of 10 or 25.

As children develop a conceptual understanding using the model, they may see the relationship between the model and how the common denominator can be calculated. This will also help the children establish the link between multiplication and division, and fractions.

Children should recognise equivalence between fractions.

Example:

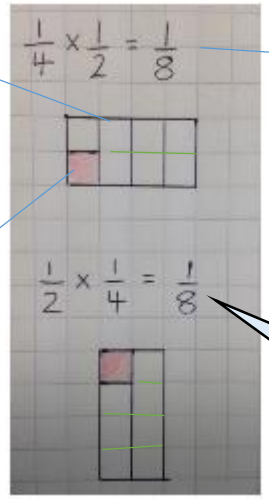


Videos: (Y4 but can be adapted to suit)

Adding fractions with different denominators (shows how to introduce the array model)
http://vimeo.com/83486433
Children exploring addition of fractions with different denominators
http://vimeo.com/83486338
Identifying Misconceptions
http://vimeo.com/83486435

Calculation of Fractions (multiplication and division) – Year 6

Key Vocabulary: fraction, whole-one/s, half, quarter, three-quarters, numerator, denominator, equal parts of a whole, mixed number, equivalence, equivalent, array, model, improper fraction, common denominator, multiples, multiply, divide, divisor, dividend, scale up/down, …of,

Objectives	Representations	Problem Solving
Multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g. $^1/_4 \times {}^1/_2 = {}^1/_8$) Multiply one-digit numbers with up to two decimal places by whole numbers (see Year 6 Multiplication) Divide proper fractions by whole numbers (e.g. $^1/_3 \div 2 = {}^1/_6$)	**Multiplication** The same array model can be used as for addition but only one grid needs to be drawn. See the example below: The example shows the multiplication carried out in both ways, ¼ x ½ and also ½ x ¼. Step 1: draw a grid that shows both quarters and halves (see addition example for Y5). At this point, only draw lines that show quarters of the grid because this is what is being multiplied. Step 2: Look at one of the quarters and shade half of it (because we are multiplying by a half). At this point children will need to know that when multiplying by a fraction, the number does not get bigger, it actually gets smaller. Some explanation or demonstration could be given to show that anything less than 1 lot of a number makes the number smaller.  Step 3: Find the fraction that is shaded by drawing an imaginary line across the whole grid halving it also. How many equal parts are shaded? (Numerator) How many equal parts has the grid been divided into? (Denominator) In this example, the answer is 1/8. To help children gain an understanding talk about 'of', for example, ½ of ¼. Language such as scaling up and scaling down can be used.	solve problems which require knowing percentage and decimal equivalents of $^1/_2$, $^1/_4$, $^1/_5$, $^2/_5$, $^4/_5$ and those with a denominator of a multiple of 10 or 25.

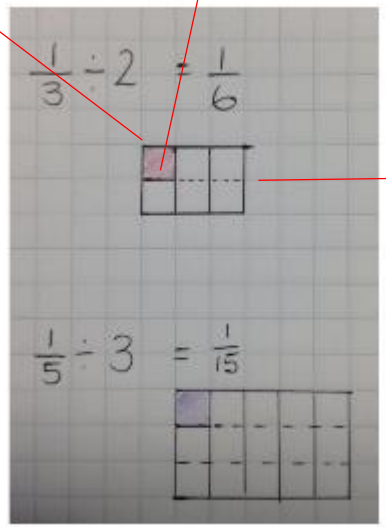
Division

Again, use a model very similar to multiplication.
Example:

Step 1: draw a grid that shows the dividend (1/3 in this case) and also the divisor (2 in this case). Draw lines down to show the dividend (1/3) but do not draw lines across to show the divisor at this point.

Step 2: look at one of the thirds and divide it by 2 (the divisor). Shade in one part of this.

Step 3: To find the answer, look at the fraction that is shaded by drawing an imaginary line across the whole grid, dividing it by 2. How many equal parts are shaded? (Numerator) How many equal parts has the grid been divided into? (Denominator). In this case the answer is 1/6



Some children will begin to understand how the fractions are calculated without the need of the model. These children will develop a deeper understanding of the relationship between multiplication, division and fractions.